Total No. of printed pages = 8

Sc-202/Maths-II/2nd Sem/2018/M

MATHEMATICS – II

(New Course)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

1. Fill in the blanks : $1\times15=15$

(i) A function f(x) is said to be —— if f(–x) = f(x) for all x.

(ii) $f(x) = 2x^2 - 3x - 1$ then $f(-2) =$ ——.

(iii) $f(x) = x+2$, $g(x) = \frac{x^2-4}{x-2}$, f(x) and g(x) are —— same functions.

(iv) Domain of $f(x) = \frac{1}{x(x+2)}$ is ——.

[Turn over

(v) $\lim_{x \to 0} \frac{\sin 2x}{3x}$ = ——.

(vi) If a function f(x) is continuous at a point a then

$f(a) = \underset{x \to a}{Lt} + f(x)$ = ——.

(vii) If y = f(x) then $\underset{h \to 0}{Lt} \frac{f(x+h) - f(x)}{h}$ = ——.

(viii) If $f(x) = \log (2x+3)$ then $f'(0)$ = ——.

(ix) A ball travels s ft. in t second where $s = 8t - 10t^2$. The velocity of the ball when t = 2 is ——.

(x) The maximum value of a function at some point may be —— —— its minimum value at some other point.

(xi) $\int x^n dx = \frac{x^{n+1}}{n+1}$ for n ——.

(xii) If derivative of a function $f(x) = x^{3/2}$ then f(x) = ——.

(xiii) In $\int \frac{dx}{x \log x}$ the substitution should be ——.

(xiv) $\int_a^b f(x)\,dx =$ ——.

(xv) Representation of the area bounded by the straight line $y = 2x$ above x-axis from $x = 1$ to $x = 4$ in the form of definite integral is ——.

2. Find the correct answer : $1\times5=5$

(a) The equation of the circle having centre at origin and radius 4 unit is

(i) $x^2+y^2 = 12$

(ii) $x^2+y^2 = 16$

(iii) $x^2+y^2 = 4$

(iv) None of these

(b) If in a coneic eccentricity (=e) = 1 then it is called a

(i) Circle (ii) Ellipse

(iii) Parabola (iv) Hyperbola

(c) The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is

(i) $\frac{2}{5}$ (ii) $\frac{3}{5}$

(iii) $\frac{4}{5}$ (iv) None of these

(d) $ax^2+2hxy+by^2+2gx+2fy + c = 0$ may represent

(i) a pair of straight line

(ii) a circle

(iii) a parabola

(iv) all the above

(e) The direction cosines of x-axis are

(i) 1, 0, 0 (ii) 0, 1, 1

(iii) 0, 0, 1 (iv) 0, 1, 1

3. Answer the following : $1\times5=5$

(a) If $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{b} = -2\hat{i} + 4\hat{j} - 3\hat{k}$, find $\vec{a} + \vec{b}$.

(b) If $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, find $\vec{a} \times \vec{b}$.

(c) If the dot product of two vectors are 'zero' then what is the angle between them ?

(d) If A = (3, 2, –5), B = (1, –1, 4), find $\overline{AB}$.

(e) Find the value of λ so that the two vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = -4\hat{i} - 6\hat{j} + \lambda\hat{i}$ are parallel.

PART – B

4. Differentiate (any *five*) : 5×3=15

(a) $y = \dfrac{(x+1)^3}{x}$

(b) $y = \log(x^2 + 3)$

(c) $y = \tan^{-1}\left(\dfrac{2x}{1+x^2}\right)$

(d) $y = \dfrac{\sin x}{\sin x + \cos x}$

(e) $y = (x)^{10}\ (10)^x$

(f) $x = 3\cos\theta - 2\cos^3\theta,\ y = 3\sin\theta - 2\sin^3\theta$

(g) $x^3 + y^3 = 3axy$

(h) $y = \sin(x)^2$

5. Answer any *one* question : 1×5=5

(a) Find the maximum and minimum value of the function

$y = x^3 - 9x^2 + 15x + 10$

Or

(b) Find the equation of tangent to the curve

$x^2 + y^2 - 6x - 8y - 1 = 0$ at $(2, -1)$

6. Evaluate any *three* : 3×3=9

(a) $\int \frac{\sin(\log x)}{x} dx$

(b) $\int \tan^2 x \, dx$

(c) $\int xe^x \, dx$

(d) $\int \frac{x^2 dx}{x^2 - 4}$

(e) $\int_0^1 \tan^{-1} x \, dx$

(f) $\int_0^{\pi/4} \sin^4 x \, dx$

7. (a) Prove that 4+2=6

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}\, dx}{\sqrt{\sin x}+\sqrt{\cos x}} = \pi/4$$

Or

Find the area of a circle $x^2+y^2=9$ using definite integral.

(b) Use the concept of odd and even functions to evaluate

$$\int_{-1}^{1} x(1-x^2)\, dx$$

8. Answer any *two* questions : 3+3=6

(a) Find the equation of the circle passing through (1,1), (2, –1), (3, 2).

(b) Find the focus, length of latus rectum, equation of directrix of the parabola $y^2 = px$, which passes through (1, 2).

(c) Find eccentricity and length of latus rectum of the ellipse $x^2 + 2y^2 = 3$.

9. Answer any *one* question : 2+2=4

(a) (i) If the position vectors of A and B are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$ respectively, find $\overline{AB}$ and determine its direction cosines.

(ii) If a line makes angles α, β, γ with the axes, prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Or

(b) (i) Find the resultant of the vectors

$$\bar{r}_1 = 2\hat{i}+4\hat{j}-5\hat{k},\ \bar{r}_2 = \hat{i}+2\hat{j}+3\hat{k}$$

(ii) Find the ratio in which the line segment joining the points (1, 2, 3) and (4, 6, –5) is divided by XOY plane.