Total No. of printed pages = 7

END SEMESTER EXAMINATION – 2019

(Regular)

Semester : 2nd (New)

Subject Code : Sc-202

MATHEMATICS – II

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Instructions :

1. *All* questions of PART–A are compulsory.
2. Answer any *five* questions from PART–B.

PART – A

Marks – 25

1. Fill in the blanks : 1×10=10

(a) The length of the diameter of the circle $x^2 + y^2 - 4x - 6y + 4 = 0$ is _____.

(b) The distance of the point (0, –4, 4) from the origin is _____.

[Turn over

(c) If two non-zero vectors are perpendicular to each other, then their scalar product is ____.

(d) If $f(x) = \sec x + \cos x$, then $f(x)$ is an ____ function.

(e) $\lim_{x \to -1} \frac{x+1}{x+2} =$ _____.

(f) At $x = 1$, the function given by

$$f(x) = x, \; x < 1$$

and $f(x) = 2x - 1, \; x \geq 1$ is _____.

(g) If $f(x) = 2x^2 + 3x - 4$, then the value of $f'(2)$ is ____.

(h) $\int \left(3x + \frac{1}{x} + 4\right) dx =$ _____ + C.

(i) $\int \frac{1}{x(\log x)^n} dx =$ _____ + C.

(j) The area of region bounded by the curve $y = 3x^2 + x$, the x-axis and the ordinates at $x = 1$ and $x = 3$ is _____.

2. Write true or false : 1×10=10

(a) The focus of the parabola $y^2 = 16x$ is (4, 0)

(b) The distance between the points (–4, –2, 3) and (3, 3, 5) is $\sqrt{78}$

(c) The direction cosines of the x-axis are 0, 0,1

(d) The domain of the function $f(x) = \frac{1}{2x-1}$ is $\left\{x : x = \frac{1}{2}\right\}$

(e) $\lim_{\theta \to 1} \frac{\sin\theta}{\theta} = 1$

(f) The left hand limit of a function f(x) at $x = 1$ is given by $\lim_{x \to 1+} f(x)$

(g) $\frac{d}{dx}(\tan^2 x) = \sec^2 x$

(h) $\int K f(x)\, dx = K + \int f(x)\, dx$

(i) $\int_0^{\pi} x \sin x\, dx = \pi$

(j) $\int \frac{dx}{1+x^2} = \tan^{-1} x$

3. Choose the correct answer : 1×5=

(a) The length of the major axis of the ellip $9x^2 + 4y^2 = 36$ is

(i) 2 (ii) 4

(iii) 6 (iv) 8

(b) If l, m, n are the direction cosines of a lir then $l^2 + m^2 + n^2$ is equal to

(i) 0 (ii) 3

(iii) 1 (iv) None of the abo

(c) The function $f(x) = \frac{1}{x-4}$ is not defined f the value

(i) 0 (ii) 1

(iii) ∞ (iv) 4

(d) $\lim_{x \to 0} \frac{\sin(x^2)}{x}$ is

(i) 0 (ii) 1

(iii) x (iv) None of the abo

(e) $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx =$

(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{3}$

(iii) $\frac{\pi}{2}$ (iv) $\frac{3\pi}{4}$

PART – B

Marks – 45

4. (a) Find the centre and radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$. 2

(b) Find the equation of the parabola with vertex (0, 0), passing through (2, 3) and axis is along the x – axis. 3

(c) In what ratio does the origin divides the line segment joining the points

P (–1, –2, –3) and Q (4, 8, 12) ? 2

(d) Find the direction cosines of the line whose direction ratios are 2, –4, 6. 2

5. (a) If $g(x) = 2^x$, show that $g(a).g(b) = g(a+b)$ 1

(b) Find limit : $\lim_{x\to 2} \frac{x^2-9x+14}{x^2+9x-22}$ 3

(c) Examine the continuity of f(x) at

$x = 0$ if $f(x) = \frac{\sin 2x}{2x}, x \neq 0$

$= 2, \quad x = 0$ 3

(d) Find $\frac{dy}{dx}$: $y = \sqrt{1+x^2}$ 2

6. (a) Find : $\frac{dy}{dx}$:

(i) $x = \frac{2at}{1+t^2}, y = \frac{a(1-t^2)}{1+t^2}$ 3

(ii) $xy = \sin(x+y)$ 3

(b) Find the maximum and minimum values of the function

$y = 4x^3 - 15x^2 + 12x - 2$ 3

7. (a) Find the unit vector in the direction of the vector $\vec{a}+\vec{b}$ if $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} + 5\vec{k}$ 3

(b) Integrate : $\int \text{cosec}x\,(\text{cosec}x + \cot x)\,dx$ 2

(c) Integrate : $\int x(3x^2+7)^7\,dx$ 3

(d) If $(x+y, x-y) = (1, 2)$, find x and y 1

8. (a) Find the value of $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x dx}{1+\tan 2x}$. 3

(b) Using the properties of definite integral, prove that 3

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$$

(c) Find the area of the region bounded by the parabola $y^2 = 4ax$ and the line $x = a$. 3

9. (a) Evaluate : 3

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots\ldots\ldots+\frac{1}{3n}\right]$$

(b) Given that $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$, $\vec{b} = 3\vec{i} - 6\vec{j} + 2\vec{k}$ and $\vec{c} = 6\vec{i} + 2\vec{j} - 3\vec{k}$, show that $\vec{a} \times \vec{b} = 2\vec{c}$. 3

(c) If $y = Ae^{mx} + Be^{-mx}$, show that $\frac{d^2y}{dx^2} - m^2y = 0$. 3