Total No. of printed pages = 10

Sc-102/Maths-I/1st Sem/Retest/2018(Old)/M

MATHEMATICS – I

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

1. Choose the correct answer : $1 \times 15 = 15$

(i) If $\log (a+3) = \log a + \log 3$ then $a =$

(a) $\frac{5}{2}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{3}{2}$

(ii) If the roots of the equation $9x^2 + 16 = Kx$ are equal, then the value of K is

(a) 6 (b) 12

(c) –10 (d) –24

[Turn over

(iii) The sum of real part and imaginary part of $\frac{(1+i)^2}{3-i}$ is

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{1}{10}$

(iv) The co-efficient of $x^{10}y^8$ in the expansion of $(x + y)^{18}$ is

(a) $^{18}C_{10}$ (b) 2^{17}

(c) $^{18}C_8$ (d) $^{18}P_{10}$

(v) If $A^2 + B^2 \propto A^2 - B^2$ then

(a) AB = constant

(b) A + B = constant

(c) A − B = constant

(d) $A \propto B$

(vi) In $(x + y)^n$, general term is

(a) t_r (b) t_n

(c) t_{n+1} (d) t_{r+1}

(vii) In ${}^{n}C_{r}$

(a) $n \leq r$ (b) $n = r$

(c) $n < r$ (d) $r \leq n$

(viii) In an A.P series a, a+d, a + 2d ..... t_n = a + (n – 1)d

(a) n = Natural number

(b) n = Integer

(c) n = Real number

(d) n = Rational number

(ix) 1 + 3 + 5 + ... to n terms =

(a) n^2 (b) $\frac{n(n+1)}{2}$

(c) $\frac{n(n-1)}{2}$ (d) $\frac{n^2-1}{2}$

(x) For what value of 'r' $a + ar + ar^2 +$ .....to ∞ is valid ?

(a) r = 1 (b) r = 0

(c) r < 1 (d) r > 1

(xi) The value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} =$

(a) 1 (b) 0

(c) 1/2 (d) None of these

(xii)(a) $e^x = 1 - x + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - \ldots\ldots$ to ∞

(b) $e^x = 1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \ldots\ldots$ to ∞

(c) $e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} + \ldots\ldots$ to ∞

(d) None of these

(xiii) Under what condition $(1+x)^n$, n any integral index is valid ?

(a) $x < 1$ (b) $x > 1$

(c) $x = 1$ (d) $|x| < 1$

(xiv) $\left|\frac{3+4i}{4-3i}\right| =$

(a) i (b) 1

(c) 7 (d) None of these

(xv) Log (1+2+3)

(a) log 1.log 2.log 3

(b) log 1+ log 2 + log 3

(c) log 2.log 3

(d) None of these

2. Choose the correct statement : $1\times5=5$

(a) (i) $\sin 2A = 2 \sin A \cos A$

(ii) $1 - \cot^2 x = \text{cosec}^2 x$

(iii) $\sin^2 x - \cos^2 x = 1$

(iv) $\cos^2 x - \sin^2 x = 1$

(b) (i) $-1 \leq \cos x < 2$

(ii) $-1 \leq \cos x < 1$

(iii) $\tan x \leq 1$

(iv) $-1 \leq \tan x \leq 1$

(c) $\tan 17^\circ + \tan 28^\circ + \tan 17^\circ \tan 28^\circ$ is

(i) 1 (ii) -1

(iii) 0 (iv) None of these

(d) $\cos x + \sin x - 1 = 0$ then $\sin 2x$ is

(i) 0 (ii) -1

(iii) 2 (iv) 1

(e) $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right)$ is

(i) π (ii) $\pi/2$

(iii) $\pi/4$ (iv) None of these

3. Find the correct answer : $1 \times 5 = 5$

(a) The perimeter of a regular pentagon is 20 cm. The length of each side is

(i) $\frac{10}{3}$ cm (ii) 4 cm

(iii) 6 cm (iv) None of these

(b) If the perimeter of a regular hexagon is 12 cm, then the area of the hexagon (in cm^2) is

(i) $\sqrt{3}$ (ii) $10\sqrt{3}$

(iii) $5\sqrt{3}$ (iv) $6\sqrt{3}$

(c) The base of a prism is a triangle. The altitude and base of the triangle is 4ft and 5ft respectively. If the height of prism be 9ft, the volume is

(i) 10 cft (ii) 25 cft

(iii) 90 cft (iv) None of these

(d) The ratio of the volumes of a cylinder and a coke on same base and of same height is

(i) 2:1 (ii) 1:3

(iii) 3:1 (iv) None of these

(e) The surface area of a sphere with radius 'r' is

(i) $4/3\ \pi r^3$ (ii) πr^2

(iii) $4\pi r^2$ (iv) None of these

PART – B

4. Answer any *four* questions : $3\times4=12$

(i) If w is the imaginary cube root of unity, prove that $(1-w+w^2)^5 + (1+w-w^2)^5 = 32$.

(ii) Find how many words can be made by using all the letters of the word DAUGHTER.

(iii) From 12 non-collinear points, how many triangles can be drawn ?

(iv) Find a quadratic equation whose roots are reciprocal of the roots of $ax^2 + bx + c = 0$.

(v) Find the term independent of x in $\left(2x^2 - \frac{1}{x}\right)^{12}$.

(vi) In an A.P series $t_{12} = 68$ and $t_{15} = 86$, find t_{18}.

5. Answer any *two* questions : $4 \times 2 = 8$

(i) Find the sum $3 + 33 + 333 + \ldots$ to ∞

(ii) Solve by Cramer's rule

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

(iii) If x, y, z are in G.P, prove that log x, log y, log z are in A.P.

(iv) Prove that $\frac{1}{\lfloor 1} + \frac{2}{\lfloor 2} + \frac{3}{\lfloor 3} + \frac{4}{\lfloor 4} + \ldots = e$.

6. Prove any *four* questions : $3\times4=12$

(i) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy}\right)$

(iii) $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$
if $A + B + C = 180^\circ$

(iv) $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = 3/16$

(v) Express $\sin 80^\circ - \sin 70^\circ$ as a product.

7. $2 \cos^2 x - 3 \sin x = 0$. 3

8. Answer any *two* questions : $5\times2=10$

(i) After flood havoc a river-side land is under erosion and appears like a curvilinear figure whose ordinates measure 9·5, 11, 13, 12·6, 10·4, 13, 15·5, 17, 14·9, 12·6, 7·7 m with base 7.3m. Apply Simpson's rule to find the approximate area of the land.

(ii) If the lateral surface area of a square pyramid is 72 ft^2 and the base edge is equal to the slant height, what is the length of the base edge ?

(iii) Find the surface area of a frustum of a right circular cone with slant height of 30m, lower base radius 20m and top radius of 15m.