Total No. of printed pages = 10

Sc-102/Maths-I/1st Sem/Com/2017/N

MATHEMATICS - I

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

1. Choose the correct answer : $1 \times 15 = 15$

   (a) $x \propto y$, $x = 2$ when $y = 9$ implies $x = 6$ when y is

   (i) 4 (ii) 11

   (iii) 7 (iv) 27

   (b) Modulus of $\frac{2+i}{4-3i}$ is

   (i) $\frac{1}{5}$ (ii) $\frac{1}{\sqrt{5}}$

   (iii) $\sqrt{5}$ (iv) 5

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(c) If $\log_a 1728 = 6$, then base a is

(i) $2\sqrt{3}$ (ii) $3\sqrt{2}$

(iii) $\sqrt{5}$ (iv) None of the above

(d) Value of ω^{16} is

(i) -1 (ii) 1

(iii) ω (iv) ω^2

(e) $\arg(2 - i2\sqrt{3})$ is

(i) π (ii) $\pi/2$

(iii) $\pi/3$ (iv) $-\pi/3$

(f) If $t_n = 292$ in 1, 4, 7, ... then n is

(i) 11 (ii) 68

(iii) 98 (iv) 97

(g) Sum of first 15 terms in $5 + 8 + 11 + ...$ is

(i) 490 (ii) 390

(iii) 460 (iv) None of the above

(h) 6th term of 2, 8, 32, ... is

(i) 563 (ii) 2408

(iii) 4902 (iv) 2048

(i) ${}^nP_r$ is calculated as

(i) $\frac{n!}{n!r!}$ (ii) $\frac{n!}{r!}$

(iii) $\frac{n!}{(n-r)!}$ (iv) $\frac{n!}{n!(n-r)!}$

(j) Number of ways that the letters of the word COLLEGE be arranged is

(i) 1000 (ii) 789

(iii) 1260 (iv) 1350

(k) Expansion of $(1 - x)^{-1}$ is

(i) $1 - x + x^2 - x^3 + x^4 - \ldots\ldots$ to infinity

(ii) $1 + x + x^2 + x^3 + x^4 + \ldots\ldots$ to infinity

(iii) $1 - x + x^2 - x^3 + x^4 - \ldots\ldots + x^n$

(iv) $1 + x + x^2 + x^3 + x^4 + \ldots\ldots + x^n$

(l) Middle term of $\left(1+\frac{1}{x}\right)^{10}$ is

(i) ${}^{10}P_4$ (ii) ${}^{10}P_5$

(iii) ${}^{10}C_5$ (iv) None of the above

(m) The general term in the binomial expansion of $(x - x^2)^{10}$ is

(i) ${}^{10}C_r x^r (-x^2)^{10-r}$ (ii) ${}^{10}C_r x^{10-r}(-x^2)^r$

(iii) $(-1)^{r10}\ C_r x^{10-r}$ (iv) ${}^{10}C_r x^{10+r}$

(n) Cofactor of a_{22} in $\begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & 1 \\ 4 & 3 & -1 \end{vmatrix}$ is

(i) $\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$ (ii) $-\begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$

(iii) $-\begin{vmatrix} -1 & 0 \\ 3 & -1 \end{vmatrix}$ (iv) $\begin{vmatrix} 2 & 0 \\ 4 & -1 \end{vmatrix}$

(o) Value of $\begin{vmatrix} 2 & 4 & 0 \\ 1 & 2 & 1 \\ 4 & 8 & -1 \end{vmatrix}$ is

(i) 1 (ii) 4

(iii) -1 (iv) 0

2. Choose the correct statement : $1 \times 5 = 5$

(a) (i) $\sin^2 x - \cos^2 x = 1$

(ii) $\sec^2 x + \tan^2 x = 2$

(iii) $\operatorname{cosec}^2 x - \cot^2 x = 1$

(b) (i) $-1 < \sin x < 1$

(ii) $\sin x \leq 1$

(iii) $-1 \leq \sin x \leq 1$

(c) (i) $\sin C + \sin D = 2\sin \frac{C+D}{2} \cos \frac{C-D}{2}$

(ii) $\sin C - \sin D = \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

(iii) $\cos C + \cos D = 2\sin \frac{C+D}{2} \sin \frac{C-D}{2}$

(d) (i) $\tan (a+b+c) =$

$$\frac{\tan a + \tan b + \tan c - (\tan a \tan b + \tan b \tan c + \tan c \tan a)}{1 + \tan a \tan b \tan c}$$

(ii) $\tan (a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

(iii) $\cot (a + b) = \frac{\cot a + \cot b}{1 + \cot a \cot b}$

(e) (i) $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$

(ii) $\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(iii) $a = b\cos C - c\cos B$

3. Find the correct answer : $1 \times 5 = 5$

(a) The cost of digging a pit of size $2 \times 2 \times 4$ at the rate of Rs. 100 is

(i) Rs. 4000 (ii) Rs. 2000

(iii) Rs. 1500 (iv) Rs. 1600

(b) The length of the longest rod that can be kept in a box of size $3 \times 12 \times 4$ is

(i) 7.9 (ii) 8.2

(iii) 12.5 (iv) 13

(c) The surface area of a sphere of radius 3 is

(i) 27π (ii) 36π

(iii) 40π (iv) 12π

(d) The base of a pyramid is a regular hexagon of side 7. If the height of the pyramid is 24 cm, its volume is

(i) 604 (ii) $417\sqrt{3}$

(iii) $538\sqrt{3}$ (iv) $588\sqrt{3}$

(e) The height of a cylinder is 10 cm and the ratio to its volume to the lateral surface area is 3 : 2. The radius is

(i) 4 (ii) 3.1

(iii) 3 (iv) 2.8

PART – B

4. Answer any *six* questions : 2×6=12

(a) (i) Transform $(2 + i)(2 - 3i)(4 - 3i)$ to $A + iB$ form.

(ii) Find square root of $7 - 24i$.

(iii) If $x + y \propto x - y$, then prove that $x^3 + y^3 \propto x^3 - y^3$.

(iv) Prove that

$$\log\frac{14}{15} + \log\frac{28}{27} + \log\frac{405}{196} = \log 2$$

(v) Find the coefficient of x^{10} in the expansion of $\left(2x^2 + \frac{1}{x}\right)^{20}$

(vi) How many odd numbers of 5 distinct significant digits can be formed with 0, 1, 2, 3, 4 ?

(vii) Prove that

$$^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$

(viii) Prove that

$$\frac{1}{1!}+\frac{3}{2!}+\frac{5}{3!}+\ldots\ldots=e+1$$

(ix) Find the sum $1 + 3 + 9 + 27 + \ldots\ldots$ to 11th term.

(x) Insert 3 AMs between 5 and 17.

5. Answer any *two* questions : $4\times2=8$

(i) Solve by Cramer's rule :

$x + 2y - z = 3$

$3x - 4y + 5z = 7$

$7x - y + z = 14$

(ii) Simplify : $\left(x+\sqrt{1-x^2}\right)^4+\left(x-\sqrt{1-x^2}\right)^4$

(iii) If $\frac{1}{x+y}, \frac{1}{2y}, \frac{1}{y+z}$ are in AP, prove that x, y, z are in GP.

(iv) If the roots of the equation $x^2 + px + q = 0$, are in the ratio 1 : 4, prove that $4p^2 = 25q$.

6. Prove any *three* : $3\times3=9$

(i) $\cos 480^\circ \sin 510^\circ - \sin 690^\circ \cos 390^\circ = 0$

(ii) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$

(iii) $\cos 3A = 4\cos^3 A - 3\cos A$

(iv) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

(v) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

7. If $A + B + C = 180^\circ$, prove that 3

$$\cos 2A + \cos 2B + \cos 2C = -4\cos A \cos B \cos C - 1$$

Or

$$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

8. Solve for θ $2\cos^2\theta + \sin\theta - 1 = 0$ 3

9. Answer any *two* questions : $5\times2=10$

(i) Find the cost of carpeting a room of triangular shape of side 8m, 6m and 10m at the rate of Rs. 200 per square meter.

(ii) The cross-section of a solid (area square cm) at a distance x from the end is as follows :

X :	0	10	20	30	40	50	60	70	80	90	100
A :	105	124	122	134	131	129	127	116	121	114	101

Applying Simpson's one-third rule, find the volume of the solid.

(iii) Find the cubical content of a pillar with base a regular hexagon of side 30 cm. What is its lateral surface area ?

(iv) A tent is made in the form of a conic frustum surmounted by a cone. The diameter of the base and top of the frustum are 20m and 6m respectively and the height is 24m. If the height of the tent is 28m, find the quantity of the canvas required to make the tent.