Total No. of printed pages = 5

Sc-102/Maths-I/1st Sem/Comm/2017/M

MATHEMATICS – I

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

(Algebra)

1. Answer any *seven* questions : $7 \times 2 = 14$

   (a) Prove that $\sqrt{i} + \sqrt{-i} = \sqrt{2}$

   (b) If $x + y \alpha x - y$, prove that $x \alpha y$.

   (c) Insert 3 Arithmetic Mean between 3 and 23.

   (d) Find the base of the logarithm of 1728 is 6.

   (e) Find the value of $x^3 - 2x^2 - 7x + 8$, when $x = 2 + \sqrt{3}$.

[Turn over

(f) Find the modulus of $\frac{12+5i}{24+7i}$

(g) Write down the 6th term of $(3x + \frac{1}{2}y)^9$

(h) If x varies directly as y and inversely as z and x = a, when y = b, z = c. Find the value of x, when $y = b^2$, $z = c^2$.

2. Prove that :

(a) $x^{\log y - \log z} \times y^{\log z - \log x} \times z^{\log x - \log y} = 1$

(b) If α and β be the roots of $2x^2 + 3x + 7 = 0$, find the values of $\alpha^2 + \beta^2$. $3\frac{1}{2}\times 2=7$

3. (a) If 'r' be the ratio of the roots of $ax^2 + bx + c = 0$, then prove that $\frac{(r+1)^2}{r} = \frac{b^2}{ac}$.

(b) Find the coefficient of x^{11} in $\left(x + \frac{2}{x^2}\right)^{17}$

$3\frac{1}{2}\times 2=7$

4. (a) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

(b) Using Cramer's rule, solve : $3\frac{1}{2}\times 2=7$

$x + y + z = 3$

$2x - y + 3z = 4$

$x + 2y - z = 2$

GROUP – B

Answer any *four* questions. $4\times 5=20$

5. (a) Evaluate $2\frac{1}{2}$

(i) cosec (-765°)

(ii) tan (780°).

(b) Prove that : $2\frac{1}{2}$

$$\frac{\sin(B-C)}{\cos B.\cos C} + \frac{\sin(C-A)}{\cos C.\cos A} + \frac{\sin(A-B)}{\cos A.\cos B} = 0$$

6. (a) Prove that 2

$$\cos 130^\circ + \cos 110^\circ + \cos 10^\circ = 0$$

(b) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$. 3

7. (a) Prove that

$$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta$$ 2

(b) Find the value of $\sin 18^\circ$. 3

8. (a) If $A + B + C = \pi$, prove that

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \cdot \sin B \cdot \sin C$$ 2

(b) The angle of elevation of the top of a tower at a distance of 400 metres from its foot is 60°. Find the height of the tower. 3

9. (a) Prove that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \pi/4$ 2

(b) In any triangle, prove that 3

$$\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cdot \cot \frac{A}{2}$$

GROUP – C

10. Answer any *three* from the following : $3 \times 5 = 15$

(a) Find the volume and curved surface of a right circular cone whose height is 8 cm and the radius of the circular base is 3 cm.

(b) A cylindrical oil tanker has its inner diameter 480 cm. If the length of the tanker is 6m, find the capacity of the tanker is c.c.

(c) Find by Simpson's rule the area of a curvilineal figure whose ordinates measure 18, 22, 26, 24, 20, 26, 30, 34, 28, 24 and 14 cm and whose base is 146 cm.

(d) Find the volume of the frustum of a pyramid the areas of whose ends are 32 sq.m and 20 sq.m and height is 6m.