Sc-102/Maths-I (N)/1st Sem/2018/M

MATHEMATICS – I

(New Course)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

1. Choose the correct answer : 1×5=5

(a) The value of i^{21} is

(i) −1 (ii) 1

(iii) i (iv) −i

(b) If $^nC_{20} = {}^nC_4$, then the value of n is

(i) 4 (ii) 16

(iii) 20 (iv) 24

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(c) The number of terms in $\left(x^2 + \dfrac{1}{x}\right)^{12}$ is

    (i) 12   (ii) 13

    (iii) 11   (iv) 10

(d) The value of $\log_{10} 1000$ is

    (i) 4   (ii) 1

    (iii) 3   (iv) 5

(e) If $A = \begin{bmatrix} 3 & 0 & 5 \\ 2 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 25 & 1 \\ 4 & 49 & 9 \end{bmatrix}$ and

    $C = \begin{bmatrix} 6/2 & 0 & 10/2 \\ 1 \times 2 & 4 & 1 \end{bmatrix}$ then,

    (i) $A = B$ (ii) $A = C$
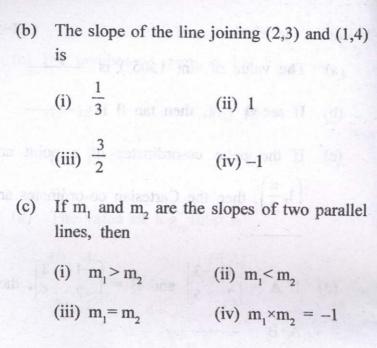
    (iii) $B = C$ (iv) $A \neq B \neq C$

2. Fill in the blanks :   1×5=5

   (a) The value of $\sin(-1305°)$ is ———.

   (b) If $\sec \theta = 1$, then $\tan \theta$ is ———.

   (c) If the polar co-ordinates of a point are $\left(1, \dfrac{\pi}{2}\right)$, then the Cartesian co-ordinates are ———.

   (d) If $A = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ -2 & -5 \end{bmatrix}$, then $A+B =$ ———.

   (e) Partial function of $\dfrac{3x-1}{x^2-1}$ is ———.

3. Choose the correct answers :   1×10=10

   (a) The co-ordinates of the middle-point of the line joining $(-2,8)$ and $(0,5)$ is

        (i) $(-2, 3)$ (ii) $(6, 5)$

        (iii) $\left(-1, \dfrac{13}{2}\right)$ (iv) $(10, 5)$

(b) The slope of the line joining (2,3) and (1,4) is

(i) $\dfrac{1}{3}$

(ii) 1

(iii) $\dfrac{3}{2}$

(iv) -1

(c) If m_1 and m_2 are the slopes of two parallel lines, then

(i) $m_1 > m_2$

(ii) $m_1 < m_2$

(iii) $m_1 = m_2$

(iv) $m_1 \times m_2 = -1$

(d) If the inclination of a straight line with the x axis is 135°, then the slope of the line is

(i) 0

(ii) ∞

(iii) -1

(iv) 1

(e) The straight line $4x - 3y + 7 = 0$ passes through

(i) (2,5)

(ii) (1,6)

(iii) (0,0)

(iv) (3,4)

(f) $\sin^{-1}\left(\sin\dfrac{\pi}{2}\right)$ is equal to

   (i)   0

   (ii)   1

   (iii) $\dfrac{\pi}{2}$

   (iv)   None of the above

(g) If $\sin\theta=\dfrac{3}{5}$, then $\tan\theta$ is

   (i) $\dfrac{5}{3}$ (ii) $\dfrac{3}{4}$

   (iii) $\dfrac{4}{5}$ (iv) $\dfrac{4}{3}$

(h) 10th term of 1, –2, 4, –8, ... is

   (i)   – 512   (ii)   19

   (iii)   512   (iv)   –17

(i) The determinant of order three contains

    (i) 2 rows 3 columns

    (ii) 3 rows 2 columns

    (iii) 3 rows 3 columns

    (iv) 2 rows 2 columns

(j) The modulus of $8 - 6i$ is

    (i) $\sqrt{28}$ (ii) 10

    (iii) $\sqrt{-2}$ (iv) -10

4. Match the following : $1 \times 5 = 5$

  (a) Area of the prism   (i) $\frac{1}{3}\pi r^2 h$

  (b) Area of the pyramid (ii) Area of the base × height

  (c) Volume of a prism (iii) Perimeter of the base × height

  (d) Volume of a pyramid (iv) 1/2 × perimeter of the base × slant height

  (e) Volume of a cone (v) 1/3 × area of the base × height

PART – B

5. Answer any *five* questions :   2×5=10

   (i) Prove that $\dfrac{1+i}{1-i} + \dfrac{1-i}{1+i} = 0$

   (ii) Insert one arithmetic mean between 3 and 20.

   (iii) Find the value of $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} \cdots\cdots$.

   (iv) How many numbers can be formed by using 4 out of the digits $1, 3, 5, 6, 7, 8, 9$?

   (v) Find the base if the logarithm of 324 is 4.

   (vi) Find the value of $\begin{vmatrix} 2 & 4 & 0 \\ 1 & 2 & 1 \\ 4 & 8 & -1 \end{vmatrix}$

   (vii) Prove that $\left(1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots \cdots \cdots\right)$

   $\left(1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \cdots \cdots \cdots\right) = 1$

6. Answer any *three* questions :   3×3=9

(i) Find the co-efficient of x^4 in $\left(x - \dfrac{1}{x}\right)^{11}$

(ii) Simplify :

$$\log \dfrac{a^3 b^3}{c^3} + \log \dfrac{b^3 c^3}{d^3} + \log \dfrac{c^3 d^3}{a^3} - 3\log b^2 c$$

(iii) Solve by Cramer's rule :

$$4x - y + 2z = 10$$
$$5x + y - z = 1$$
$$3x + 3y - 2z = -7$$

(iv) If a, b, c be in A.P. and x, y, z in G.P., prove that $x^{b-c} \; y^{c-a} \; z^{a-b} = 1$

(v) If $^nP_6 = 30.\ ^nP_4$, find n

7. Answer any *three* questions :   3×3=9

(i) Show that the points (2,5), (4,6) and (8,8) lie on a straight line.

(ii) Find the equation of the straight line which passes through (–1, 4) and perpendicular to $3x + 4y + 28 = 0$

(iii) Find the length of perpendicular from the origin on the line $2x - 3y + 1 = 0$

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(iv) Show that the angle between the lines $\sqrt{3}\ y = x + 2$ and $\sqrt{3x} + 4$ is $30°$

(v) Reduce $2x + 3y - 5 = 0$ to the intercept form and find x-intercept and y-intercept.

8. Prove (any *four*) : $3×4=12$

(i) $\sin 600° \cos 330° + \cos 120° \sin 150° = -1$

(ii) $\dfrac{\sin A}{1 + \cos A} + \dfrac{1 + \cos A}{\sin A} = 2 \text{cosec} A$

(iii) $\dfrac{\sin (A+B) + \sin (A-B)}{\cos(A+B) + \cos(A-B)} = \tan A$

(iv) $\dfrac{\sin 2A}{1 + \cos 2A} = \tan A$

(v) $\dfrac{\sin 3A}{\sin A} - \dfrac{\cos 3A}{\cos A} = 2$

(vi) $\dfrac{\sin 60° + \sin 30°}{\cos 30° - \cos 60°}$

(vii) $\cos 20° \cos 40° \cos 60° \cos 80° = \dfrac{1}{16}$

9. (i) The cross sectional area of a tunnel is as follows :   3

Distance from one end :	0	3	6	9	12	15	18
Area	27.9	30.6	33.8	32.4	30.7	27.9	26.1

Find the volume of the tunnel.

(ii) The curved surface area of a cone is 4070 cm² and its diameter is 70cm. What is its slant height ?

(Use $\pi = 22/7$)   2