

2023

NUMERICAL METHODS FOR ENERGY SYSTEMS

Full Marks: 100

Time: Three hours

*The figures in the margin indicate full marks for the questions.**(Answer any five questions.)*

- 1 a) Discuss graphically how to derive Regula Falsi formula 4
- b) Using Regula Falsi method find a real root of $x^3 - 4x - 9 = 0$ correct to three decimal places. 6
- c) Using Bisection Method find a real root of $e^x - x = 2$ correct up to three decimal places. 5
- d) Using Successive Approximation Method, find a real root of $3x^3 - 9x^2 + 8 = 0$ correct to for decimal places. 5
2. a) Solve the non-linear differential equations 10
- $$2x^2 + 3xy + y^3 = 3, -4x^2 + 2xy + y^2 = 30.$$
- Correct to three decimal places using Newton-Raphson method, given that $x_0 = 0.5$ and $y_0 = -2$
- b) Consider the second order value problem $y'' = x^3y + x^2y'$ with initial condition, $y(0) = 1, y'(0) = \frac{1}{2}$. 10
- Using the fourth order Runge-Kutta method find $f(0.2)$ correct to fourth decimal places.
3. a) State and Prove Newton's Forward Interpolation formula. 10
- b) Compute (1.2) from the following data: 10

x	1.0	1.5	2.0	2.5	3.0
f(x)	9.0	32.75	79.0	155.25	269.0

4. a) Given, 10

x	1	2	3	4	5	6	7	8
F(x)	1	8	27	64	125	216	343	512

Estimate $f(7.5)$.

- b) The observed values of a function are respectively 168, 120, 72 and 63 at the positions 3, 7, 9 and 10 of the independent variable respectively. What is the best estimate you can give for the value of the function at the position 6 of the independent variable? 10

5. a) (a) The following data gives the velocity of a particle for twenty seconds at an interval of five seconds. Find the initial acceleration using the entire data 7

Time t(sec):	0	5	10	15	20
Velocity v(m/sec)	0	3	14	69	228

- b) The elevation above a datum line of seven points of a road are given below: 7

x:	0	300	600	900	1200	1500	1800
Y:	135	149	157	183	201	205	193

Find the gradient of the road at the middle point.

- c) From the table below, for what value of x, y is minimum? Also find this value of y 6

x:	3	4	5	6	7	8
Y:	0.205	0.240	0.259	0.262	0.250	0.224

6. a) Evaluate the integral $\int_0^2 e^{-x^2} dx$ using Trapezoidal Rule taking the number 10 intervals. 5

- b) Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x) dx$ using Simpson's 3/8 rule. 5

- c) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines $x = 0$ and $x = 1$ and the curve through the points with the following co-ordinates 4

x:	0.00	0.25	0.50	0.75	1.00
y:	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.

- d) Using the Runge-Kutta method of the fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$ and 0.4 . 6
