Total number of printed pages: 03

PG/1st/ PCSE102

2023

Algorithms and Algorithmic Complexity

Full Marks : 100

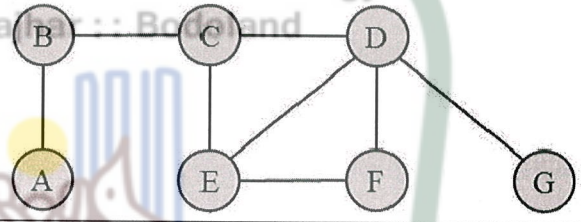
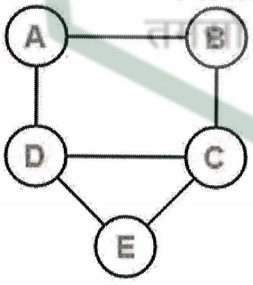
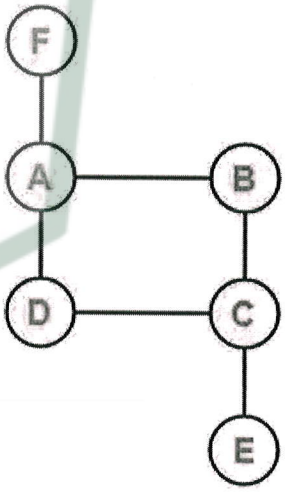
Time : Three hours

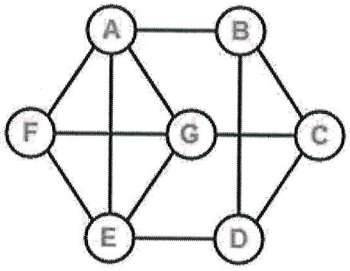
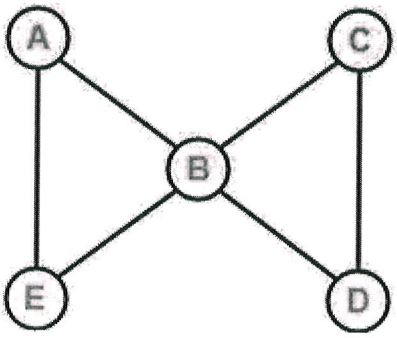
The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1.	a)	Prove that if $f(n) = a_m n^n + a_{m-1} n^{n-1} + \ldots\ldots + a_1 n + a_0$ then $f(n) = \theta(n^m)$	6
	b)	Find the asymptotic upper bound for T(n) of the following recurrence. Assume that T(1) = 1. $T(n) = T(n-1) + \frac{1}{n}$	6
	c)	Using master's theorem find the time complexity for the following recurrences using master's theorem: *(i)* $T(n)=4T(n/2)+n^2\sqrt{n}$ *(ii)* $T(n) = 16T\left(\frac{n}{4}\right)+n^2$	4+4
2.	a)	Suppose we're doing a sequence of n operations (numbered 1, 2, 3,...) on a data structure in which the ith operations cost is as follows: $cost = \begin{cases} 1 & if\ i \neq power\ of\ 2 \\ i & if\ i = power\ of\ 2 \end{cases}$ For example, the following table shows the costs for each of the first few operations: operation number: 1 2 3 4 5 6 7 8 9 ... cost: 1 2 1 4 1 1 1 8 1 ... Find the amortized cost per operation of augmented stack using potential analysis	10

	b)	Find the amortized cost per operation using aggregate analysis of the Table-Insert operation(Algorithm-1) given below:
		`Algorithm - 1: Table-Insert (T,x)`
		`1 if T.size == 0`
		`2 allocate T.table with 1 slot`
		`3 T.size = 1`
		`4 if T.num == T.size`
		`5 allocate newTable with 2*T.size slots`
		`6 insert all items in T.table into newTable`
		`7 free T.table`
			10

		8 T.table = newTable 9 T.size = 2*T.size 10 insert x into T.table 11 T.num = T.num + 1	
3.	a)	Define the classes P and NP. Discuss diagrammatically the relations among P class, NP class, NP hard and NP complete.	10
	b)	Assume that 3-SAT problem as NP-Complete problem. Reduce Clique decision problem to 3-SAT problem.	10
4.	a)	Apply the 2-approximate on the following graph for vertex cover. Also find the Approximation ratio (p(n)). 	
	b)	How does the concept of Hamiltonian circuits relate to the Traveling Salesman Problem? Determine whether following graphs are Hamiltonian graph or not. Provide a proper justification for your answer in terms of Hamiltonian cycle/circuits.  (a)  (b)	

		 (c)  (d)	
5.	a)	Illustrate the operation of Partition in the context of the quicksort algorithm on the array: A = (13,19,9,5,12,8,7,4,21,2,6,11) Assuming that the last element (that is, 11) is chosen as the pivot element, show the steps involved in one pass of the partitioning process that places the pivot element in its final position in the sorted list.	
	b)	Write the algorithm(including partition) of Quick sort	
6	a)	Illustrate the operation of buildMaxHeap(A,9) on the array A=<5,3,17,10,84,19,6,22,9> to make a max heap tree.	
	b)	Show how the **merge sort** algorithm will sort the following array in increasing order: 70,80,40,50,60,12,35,95,10	

-----xxx-----