PG(M.Tech) & PhD(Course Work)/1st Semester/PCSE102 & PCSE5102

2022

Algorithms and Algorithmic Complexity

Full Marks: 100

Time: Three hours

The figures in the margin indicate full marks for the questions.

Answer any five(5) **20×5=100**

1. (a) Define and differentiate asymptotic notations(O, Θ, Ω). (6+6+6)

(b) Draw the recursive tree for the recurrence relation:

$T(n)=T(n/3)+T(2n/3)+n$

(c) Solve the recurrence relation.
$T(n)=2T(n/2)+1$, $T(1) = 1$.

2. (a) What does dynamic programming have in common with divide-and-conquer? What is a principal difference between them? (4+6+10)

(b) Design an efficient algorithm for computing the binomial coefficient C(n, k) that uses no multiplications. What are the time and space efficiencies of your algorithm?

(c) Illustrate the operation of Partition in the context of the quicksort algorithm on the array:

A = (2, 8, 7, 1, 3, 5, 6, 4)

Assuming that the last element (that is, 4) is chosen as the pivot element, show the steps involved in one pass of the partitioning process that places the pivot element in its final position in the sorted list.

3. (a) Suppose we're doing a sequence of n operations (numbered 1, 2, 3,...) on a data structure in which the ith operations cost is as follows: [10+10]

$$cost = \begin{cases} 1 \ if \ i \neq \ power \ of \ 2 \\ i \ \ if \ i = power \ of \ 2 \end{cases}$$

For example, the following table shows the costs for each of the first few operations:

operation number:	1	2	3	4	5	6	7	8	9 ...
cost:	1	2	1	4	1	1	1	8	1 ...

Use aggregate analysis to determine amortized cost per operation.

(b) Find the amortized cost per operation of augmented stack using potential analysis

4. (a) Illustrate the operation of buildMaxHeap(A,8) on the array A=<4,1,3,2,16,9,10,14,8,7> to make a max heap tree. [10+10]

(b) Prove that the average case time-complexity of merge sort is O(n log n).

5. Show steps of Kruskal's and Prim's algorithm to find a minimum spanning tree of the graph shown in the Figure 2. [10+10]

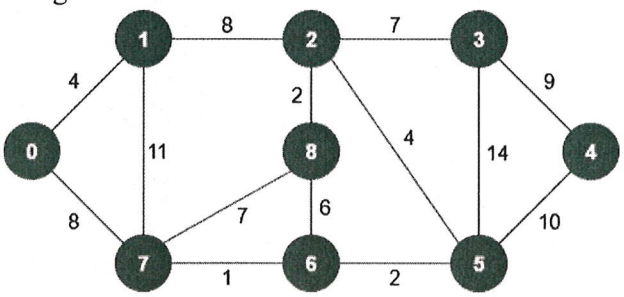


Figure 2

6. (a) Trace the steps to solve the 4-Queens problem by backtracking method. For each step draw the 4×4 matrix showing the positions of queens in it. Show where you apply backtracking. [10+10]

(b) Find out Hamiltonian cycle of the graph given in figure 4 and also draw the state space tree.

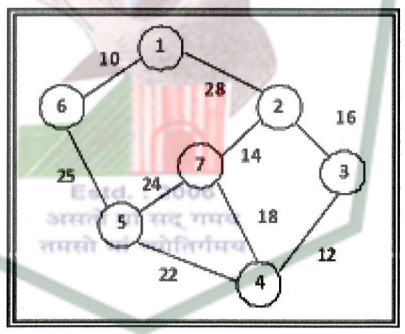


Figure 4

7. (a) Apply the 2-approximate algorithm for the Travelling Salesman Problem of the following graph (Figure 5). [10+10]

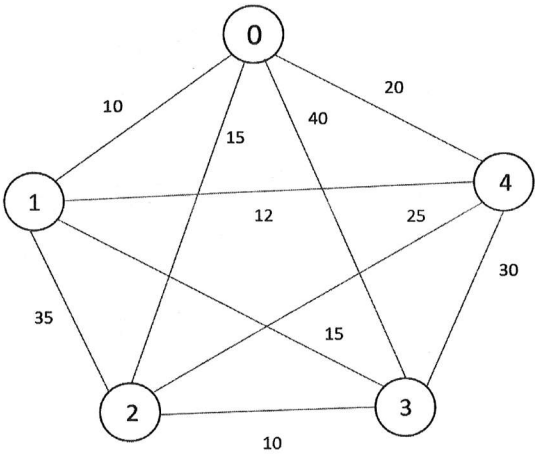
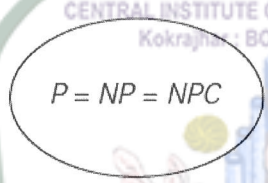


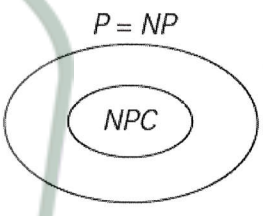
Figure 5

(b) Which of the following diagrams do not contradict the current state of our knowledge about the complexity classes *P*, *NP*, and *NPC* (*NP*-complete problems)?

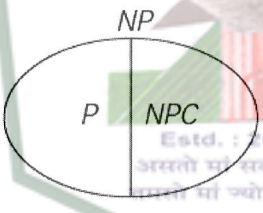
a.



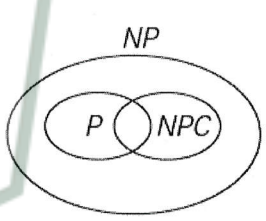
b.



c.



d.



e.