Total No. of printed pages = 13

PG/1st Sem/PCSE 102

2021

ALGORITHMS AND ALGORITHMIC COMPLEXITY

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

A. Multiple choice questions : 1×20=20

1. Every graph has only one minimum spanning tree

   (a) true

   (b) false

2. What is the correct definition of an algorithm ?

   (a) An algorithm is a step by step instructions to solve a problem.

   (b) An algorithm is a process of baking bread.

[Turn over

(c) An algorithm is a software used to compute numbers.

(d) An algorithm is the process of breaking problems.

3. What is vertex coloring of a graph ?

(a) A condition where any two vertices having a common edge should not have same color

(b) A condition where any two vertices having a common edge should always have same color

(c) A condition where all vertices should have a different color

(d) A condition where all vertices should have same color

4. Which of the following is not a type of graph in computer science ?

(a) undirected graph

(b) bar graph

(c) directed graph

(d) weighted graph

5. Prim's algorithm is a

   (a) Divide and conquer algorithm

   (b) Greedy algorithm

   (c) Dynamic programming

   (d) Approximation algorithm

6. Which of the following is false about Prim's algorithm ?

   (a) It is a greedy algorithm

   (b) It constructs MST by selecting edges in increasing order of their weights

   (c) It never accepts cycles in the MST

   (d) It can be implemented using the Fibonacci heap

7. What is the time complexity of Kruskal's algorithm ?

   (a) O(|E| log |E|)

   (b) O(E log V)

   (c) $O(E^2)$

   (d) O(V log E)

8. Backtracking algorithm is implemented by constructing a tree of choices called as

   (a) State-space tree

   (b) State-chart tree

   (c) Node tree

   (d) Backtracking tree

9. In what manner is a state-space tree for a backtracking algorithm constructed ?

   (a) Depth-first search

   (b) Breadth-first search

   (c) Twice around the tree

   (d) Nearest neighbour first

10. Which of the problems cannot be solved by backtracking method ?

   (a) n-queen problem

   (b) Graph coloring problem

   (c) Hamiltonian circuit problem

   (d) Travelling Salesman Problem

11. An algorithm is made up of two independent time complexities f(n) and g(n). Then the complexities of the algorithm is in the order of

(a) $f(n) \times g(n)$

(b) max (f(x), g(x))

(c) mx (f(x), g(x))

(d) f(n) + g(n)

12. Which of the following problems should be solved using dynamic programming ?

(a) Mergesort

(b) Binary search

(c) Longest common subsequence

(d) QuickSort

13. What is the typical running time of a heap sort algorithm ?

(a) O(N) (b) O(N log N)

(c) O(log N) (d) $O(N^2)$

14. A linear search is to be performed on the list : 12 6 8 11 13. _____ is the complexity to find number 12 ?

(a) O(n) (b) O(1)

(c) O(log n) (d) O(n log n)

15. QuickSort is

(a) Brute Force technique

(b) Divide and conquer algorithm

(c) Greedy algorithm

(d) Dynamic programming

16. What is a randomized QuickSort ?

(a) The leftmost element is chosen as the pivot

(b) The rightmost element is chosen as the pivot

(c) Any element in the array is chosen as the pivot

(d) A random number is generated which is used as the pivot

17. In what position does the array for heap sort contains data ?

(a) 0

(b) 1

(c) –1

(d) Anywhere in the array

18. Which of the following is true about NP-Complete and NP-Hard problems ?

(a) If we want to prove that a problem X is NP-Hard, we take a known NP-Hard problem Y and reduce Y to X.

(b) The first problem that was proved as NP-complete was the circuit satisfiability problem.

(c) NP-complete is a subset of NP Hard.

(d) All of the above.

19. Let X be a problem that belongs to the class NP. Then which one of the following is TRUE ?

(a) There is no polynomial time algorithm for X.

(b) If X can be solved deterministically in polynomial time, then P = NP.

(c) If X is NP-hard, then it is NP-complete.

(d) X may be undecidable.

20. Assuming P ! = NP, which of the following is true ?

(a) NP–complete = NP

(b) NP–complete ∩ P = NULL

(c) NP–hard = NP

(d) P = NP–complete.

B. Answer any *four* of the following : 20×4=80

1. Consider a fractional knapsack of capacity = 6. The profit and size of six products are listed below :

Product	P1	P2	P3	P4	P5	P6
Size	1	2	3	4	5	6
Profit	10	14	15	60	60	66

(a) Compute the optimum profit using a greedy method.

(b) Prove the correctness of your algorithm.

(c) Compute the complexity of your algorithm.

(d) Do you believe the above algorithm will also work for 0/1 knapsack problem? Justify.

(e) Describe how one can implement each of the following operations on an array so that the time it takes does not depend on the array's size n. Delete the i th element of an array $(1 \le i \le n)$.

4+3+4+3+6=20

2. (a) Compute the upper bound and tightly bound for the function $f(n) = 3n^2+15n+13$.

(b) Assume another function

$g(n) = 7n^2+13n+15$. Justify whether $g(n) = \Theta(f(n))$.

(c) Prove or disprove the followings :

(i) $f(n) = O(g(n))$, $g(n) = O(l(n))$ then $g(n) = O(f(n))$.

(ii) $f(n) = \Theta(g(n))$, $g(n) = \Theta(l(n))$ then $g(n) = \Theta(f(n))$.

(d) List the following functions according to their order of growth from the lowest to the highest :

$(n-2)!$, $5 \lg(n+100)^{10}$, 2^{2n}, $0.001n^4+3n^3+1$, $\ln^2 n$, $\sqrt[3]{n}$, 3^n. 4+4+3+3+6=20

3. (i) Consider the following processes needs to be executed by a single processor :

Process	P1	P2	P3	P4	P5	P6
Arrival time	1	2	4	3	7	5
Finish time	3	4	6	5	11	8

Process	P7	P8	P9	P10	P11
Arrival time	6	8	11	14	12
Finish time	12	12	14	15	15

(a) Use brute force approach to find out the maximum number of processes can be executed by the processor.

(b) Compute the complexity of your brute force approach.

(c) Use dynamic programming to solve the problem.

(d) Determine the complexity of your algorithm for the dynamic programming approach.

(ii) Apply mergesort to sort the list K,O,K,R,A,J,H,A,R in alphabetical order.

4+1+4+1+10=20

4. (a) Illustrate the operation of build Max-Heap (A,8) on the array A = <4,3,2,16,9, 10,14,8> to make a max heap tree.

(b) Prove that the average case time-complexity of mergesort is O(n log n).

(c) Find the average case time complexity of quicksort. 7+5+8=20

5. Show steps of Kruskal's and Prim's algorithm to find a minimum spanning tree of the graph shown in the Figure 1. 10+10=20

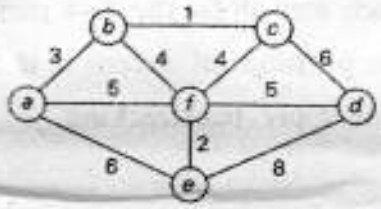

Figure 1

6. (a) Find out Hamiltonian cycle of the graph given in figure 2 and also draw the state space tree. 7+7+6=20

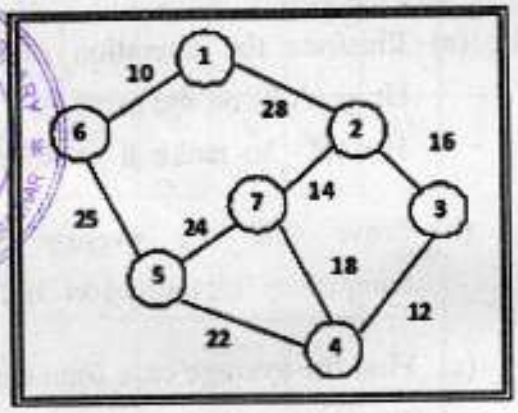

Figure 2

(b) Trace the steps to solve the 4-Queens problem by backtracking method. For each step draw the 4×4 matrix showing the positions of queens in it. Show where you apply backtracking.

(c) Consider the following recursive algorithm :

ALGORITHM *S(n)*

//Input: A positive integer *n*

Step 1. if n = 1

Step 2. **return** 1

Step 3. **else**

Step 4. **return** S(n – 1) + 2 * n – 1

Set up a recurrence relation for this algorithm and solve it. 7+7+6=20