

2023
Mathematical Foundation in Computer Science

Full Marks : 100
Time : Three hours

Answer question number 1 and any *four* from the rest

1. Answer all the questions
- i) $S = \{\{\{\}\}, \{\}\}$, the number of elements in $P(S)$ is
 - ii) A, B and C are three sets, where $|A| = |B| = |C| = 5$ and $|A \cap B| = |A \cap C| = |B \cap C| = |A \cap B \cap C| = 2$. Then $|A \cup B \cup C| = ?$
 - iii) Draw the venn diagram of $A^c - B^c$
 - iv) In a group of 15 students; 10 like mathematics and 10 like machine learning and 3 like none of these. How many of them like both?
 - v) Explain that the *friendship* relation is not transitive.
 - vi) The inverse element of $-i$ in the multiplicative group, $\{1, -1, i, -i\}$ is
 - vii) Write a recurrence relation for the sequence $1, a, a^2, a^3, a^4, ..$
 - viii) What is the criteria that a poset become lattice?
 - ix) Find the solution of the recurrence relation $t_n = at_{n-1} (t_0 = 1)$

2 x 10

2. a) Prove that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim q \rightarrow \sim p)$ are logically equivalent.
b) Given that $P(x)$: " $x+2 > x$ ", then find the truth value of $\forall P(x)$. State the difference between $\exists x \forall y P(x, y)$ and $\forall y \exists x P(x, y)$ with an example.
c) Express the statement "Every student in this class has studied MFCS" into logical expression. Write the negation of the above statement.

6 + 6 + 4 + 4

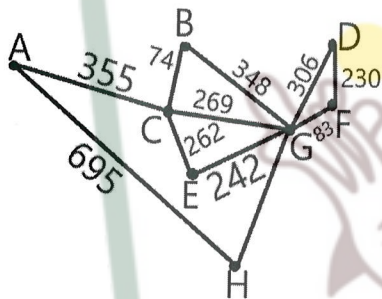
3. a) Prove that for all sets A and B, if $B \subseteq A^c$ then $A \cap B = \phi$.
 b) Prove that ${}^{n+1}C_r = {}^n C_r + {}^n C_{r-1}$
 c) Find the minimum number of students in a class such that three of them are born in the same month?

8 + 6 + 6

4. a) Define the partial order set. When a partial order set become a lattices?
 b) Draw the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$. Is this Hasse diagrams a lattice?

10 + 10

5. a) Define spanning tree. Find the minimum spanning tree of the given graph using Kruskal's Algorithm.



- b) Describe the breadth-first search algorithm. Do the depth-first search on the above graph

10 + 10

6. Write a short note on the following (any four)

- (i) Modus ponens
 (ii) Crisp set and Fuzzy set
 (iii) Hamiltonian path and Euler graph
 (iv) Tower of Hanoi problem
 (v) Recursive function for $(x + y)$

5 x 4

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