2023

Mathematical Foundation in Computer Science

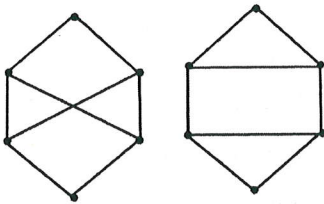*Full Marks: 100*

Time: Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1.	a)	Let p and q be the propositions defined as p: "The result is decided" and q: "The copies have been evaluated". Express each of these compound propositions as an English sentence (i) ¬q → ¬p (ii) p ↔ q	8
	b)	Prove that p ↔ q and (p ∧ q) ∨ (¬q → ¬p) are logically equivalent.	6
	c)	Given that P(x): "x+2 > x", then find the truth value of ∀P(x). Explain the difference between ∃x ∀y P(x, y) and ∀y ∃x P(x, y) with an example.	6
2.	a)	Describe the derangement problem. Establish a Recurrence Relation for the Derangement problem.	12
	b)	Solve the recurrence relation $T(n) = T(n-1) + 2n$ and $T(0) = 0$	8
3.	a)	State and prove the multinomial theorem	12
	b)	Find the term that is independent of x in the expression $\left(x^2 + x + \dfrac{1}{x}\right)^{10}$	8
4.	a)	Define the partial order set. When a partial order set become a lattices?	6
	b)	Consider the set S = {a, b, c} and P(S) is the power set of S, then show that (P(S), __) is a partial order set.	6
	c)	Draw the Hasse diagram of the above partial order set. Is it a lattices?	8
5.	a)	Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of edges. Does the theorem hold for a multigraph? Justify your answer with example?	8

	b)	Define the i) Hamiltonian path ii) Euler graph.	6
	c)	Define isomorphic graph? Verify whether the following graphs are isomorphic.	6
6.	a)	A drawer contains 12 two rupee coins and 12 five rupee coins. You take the coins randomly from the drawer. How many coins you must take out to be sure that you can buy an article of costs Rs 25?	6
	b)	Prove that $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$	6
	c)	Define cyclic group. Show that the cube roots of unity form a cyclic group with respect to multiplication operation.	8