Total No. of printed pages = 4

PG/1st Sem/PCSE 101

2021

MATHEMATICAL FOUNDATION IN COMPUTER SCIENCE

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer any *five* questions.

1. (a) Let A = {1, 2, 3, 4, 5, 6, 7} and the relation R on A × A is given by R = {(x, y)|x–y is divisible by 3}. Show that R is an equivalence relation. 10

   (b) There is a lottery in a local fair. There are 120 lottery tickets labeled the numbered 1 to 120. The cost of each ticket is Rs 50 for each ticket. 120 people buy the ticket randomly. The persons with a ticket of multiple of 7 won the first prizes, the ticket of multiple of

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5 won the second prizes and the ticket of multiple of 2 won the third prizes, and the rest have received a pen. How many pens are needed to conduct the lottery? 10

2. (a) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} such that there is at least one pair of these numbers their sum is 16? 6

(b) In how many ways can we put 4 different letters into 4 different envelopes so that all the letters go into the wrong envelopes? 6

(c) State and prove the binomial theorem. 8

3. (a) Solve the following recurrence relations:

(i) $U_n = U_{n-1} + n$ where $U_0 = 0$. 4

(ii) Write the Fibonacci sequence in recursive form and solve it. 6

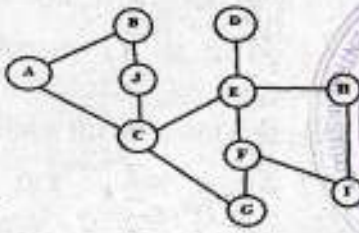
(b) Write a recursive function to convert a decimal number to a binary number. 6

(c) Give the recursive definition an arithmetic expression. 4

4. (a) Describe the depth first search and its advantages and disadvantages. 10

(b) Prove that in an undirected graph the number of odd degree vertices are even. 6

(c) Is the following graph is a bipartite graph ? 4



5. (a) Consider f(x, y) as "x + y = 1". Find the truth values of the quantifications $\exists y \forall x$ f(x, y) and $\forall x \exists y$ f(x, y), where the domain for all variables consists of all real numbers ? 8

(b) Translate the following sentence into a logical expression and do the negation of that expression. "Every BTech student of CIT must have a programming course". 8

(c) Prove that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. 4

6. (a) Define the greatest and least element of a poset. Consider the poset ({3, 5, 9, 15, 24, 45}, |). Draw the hasse diagram. Is it a lattice? Explain your answer? 12

(b) a, b are two elements of a group and it is given that $(ab)^2 = a^2b^2$, then prove that the group is abelian. 8