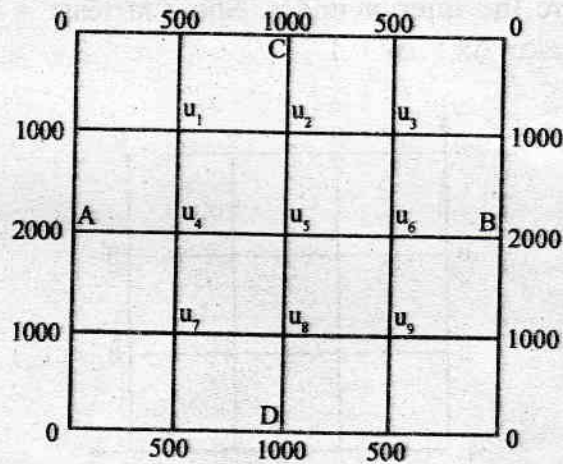**2022**

ADVANCED COMPUTATIONAL HYDRAULICS

Full Marks – 100

Time – Three hours

The figures in the margin indicate full marks
for the questions.

1. Consider a plate of size 4m × 4m that is subjected
   to the boundary condition as shown in the figure.
   Find the temperatures at the interior using suitable
   grid size. Show at least 3 iteration. Use Laplace
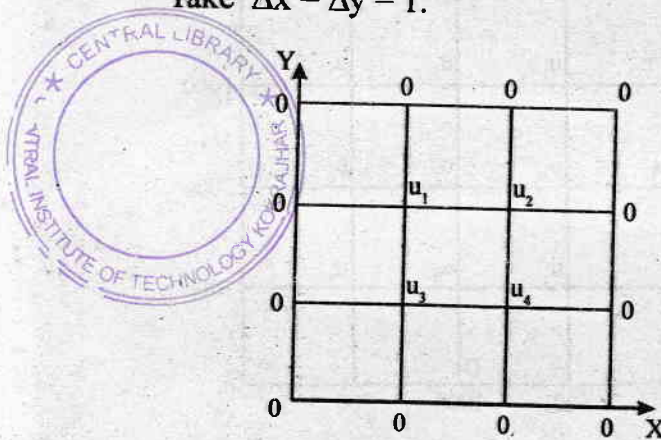   equation.   20

2. Consider a steel rod that is subjected to temperature of 125°C on the left end and 50°C on the right end. If the rod is of the length 0.1m, use finite difference method to find the temperature distribution in the rod from $t = 0$ sec and $t = 9$ sec. Use $\Delta x = .02m$ and $\Delta t = 3$ sec and $\lambda = 1$.

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3. The transverse displacement u of a point at a distance x from one end and at any time t of a vibrating string satisfies the equation $\partial^2 u/\partial t^2 = 4\partial^2 u/\partial x^2$, with boundary conditions $u = 0$ at $x = 0$, $t > 0$ and $u = 0$ at $x = 4$, $t > 0$ and initial conditions $u = x(4 - x)$ and $\partial u/\partial t = 0$, $0 \le x \le 4$. Solve this equation numerically for one-half period of vibration, taking $h = 1$ and $k = 1/2$.

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4. Solve the equation $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = -10(x^2 + y^2 + 10)$ as shown in the figure, find the values are the interior nodes. Show at least 3 iteration. Take $\Delta x = \Delta y = 1$.

(2)

5. Solve $\partial^2 y/\partial t^2 = \partial^2 y/\partial x^2$ up to $t = 0.5$ with a spacing of 0.1 subject to $y(0, t) = 0$, $y(1, t) = 0$, $y(x, 0) = 0$ and $y(x, 0) = 10 + x(1 - x)$.   20