

Central Institute of Technology Kokrajhar

Computer Science & Engineering Department

END SEMESTER EXAMINATION**Instructions:** Attempt Question No. 1 and any four from the rest.

1. a) Which one of the following two is a proposition and why?
 - i. $5+6=13$
 - ii. $x+5=13$
- b) How many subsets may be formed from the following set S -Justify
 $S = \{2, 4, 5, 7, 9\}$.
- c) Translate the following sentences in English $\forall x \exists y (x > y)$.
- d) Prove that adding two odd numbers always produces even numbers.
- e) Consider two sets A and B. Justify whether the following is true or not -
 $A \times B = B \times A$
- f) Check whether the following is true or not
 $(A \cap B') = A - B$
- g) Compute the inverse of the function $f(x) = x^2 + 1$.
- h) How many colors can be used to color a complete graph with ten vertices?
- i) Justify the following statement - *iff there is a Hamiltonian Cycle in a Graph then there exists a Hamiltonian path.*
- j) Consider the integers from 1 to 3000. How many of them are divisible by both 5 and 7?
[10× 2 =20 marks]

2. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
 - a) Someone in your class has visited Delhi.
 - b) No one in your class owns both a desktop and a laptop.
 - c) Everyone in your class either study MFCS or DAA.
 - d) Someone in your class always remains happy.
 - e) Not everybody loves DAA
 - f) If anyone has a desktop or laptop, studies MFCS, and remains happy then she/he clears the GATE examination.

[((3× 5)+5) = 20 marks]

Please Turn Over

3. a) Use contradiction to prove that if x is an integer and $x^4 + 3$ is an odd number then x is even.
 b) Suppose that on an island there are three types of people, **Wise**, **Clever**, and **Simple**. **Wise** always tell the truth, **Clever** always lie, and **Simple** sometimes lie and sometimes tell the truth. Detectives questioned three inhabitants of the island — Robin, David, and Sandy as part of the investigation of a crime. The detectives knew that one of the three committed the crime, but not which one. They also knew that the criminal was a Wise and that the other two were not. Additionally, the detectives recorded these statements-
 Robin: *I am innocent.*
 David: *What Robin says is true.*
 Sandy: *David is not a normal*
 After analyzing the available information, the detectives successfully identified the guilty party. Who was it?
 c) With an example discuss **Modus Ponens** and **Modus Tollens**.

[(5+10+5) = 20 marks]

4. a) Consider a completely connected graph $G=(V, E)$, where $V = \{v_1, v_2, \dots, v_{10}\}$, and the weights of the edges $e(i, j)$ between vertex v_i and v_j is computed as the maximum of (i, j) . What will be the weight of the minimum spanning tree?
 b) With an example discuss Graph Isomorphism.
 c) Determine whether the following function is a bijective function from the domain $R \rightarrow R$
 $f(x) = \frac{x+3}{x+1}$.

[(8+6+6) = 20 marks]

5. a) Recently Government of India has introduced the concept of **BH (Bharat)** series bike/car number. The first two digits of the number contain the last two digits of the car selling year, followed by **BH**, followed by any four numbers (each number can range from 0-9), and finally another character from **A** to **Z**. An example of such a bike/car is given below - **24BH0134Z**.
 How many license plates can be made for all the bikes/cars purchased in 2024? What will be the total number of license plates that can be generated irrespective of years?
 b) To ensure that at least six people share the same zodiac sign, how many individuals need to be present if there are 12 zodiac signs?
 c) Consider there is a group of 100 students - 50 boys and 50 girls in your class. To organize the upcoming **Ecstasy 2025**, we need to form a core committee of 10 members containing 5 boys and 5 girls students.
 How many ways the committee can be formed?
 There is also a need to identify the General Secretary, Cultural Secretary, and Sports Secretary irrespective of gender. How many ways these members can be selected from the 100 students?

[(8+5+7) = 20 marks]

6. a) Solve the following recurrence relations:
 i. $f_n = f_{n-1} + 2f_{n-2}, f_0 = 5, f_1 = 7$,
 ii. $f_n = 3f_{n-2} + 2f_{n-1} + 2^n, f_0 = 1, f_1 = 3$,
 b) Consider a recurrence relation of an algorithm as follows - $T(n) = 4T(n/4) + c$, where c is a constant. What will be the time complexity of the algorithm (in terms of big **O** notation)?
 c) Consider a set $A = a^2 + 3, a \in N$, Check whether $(A, +)$ is an Abelian Group or not ?

[((4+5)+6+5) = 20 marks]