53 (MA 401) NMCP

2021

NUMERICAL METHODS AND COMPUTER PROGRAMMING

Paper : MA 401

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* State Newton's forward interpolation formula. Compute $\sin 52°$ from the following data :

x	:	$45°$	$50°$	$55°$	$60°$
$\sin x$	:	0·7071	0·7660	0·8192	0·8660

1+7=8

(b) Write an algorithm to implement Euler's method. Solve the differential equation $\frac{dy}{dx} = x^2 + y$, with $y(0) = 1$ using Euler's method by choosing a step size of $h = 0{\cdot}2$ to obtain solution for $0 \le x \le 1$.

4+8=12

Contd.

2. *(a)* Solve $x^3 + 4\sqrt{x} - 3 = 0$, correct to 3 places of decimal by Bisection Method. 8

(b) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = x^3 + \frac{1}{2}y$ with $y(1) = 2$ at $x = 0{\cdot}1,\ 0{\cdot}2,\ 0{\cdot}3$. 12

3. *(a)* Fit a parabola $y = a + bx + cx^2$ to the following data :

x :	2	4	6	8	10	12
y :	3·07	12·85	31·47	57·38	91·29	100·02

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(b) Prove that : 2½×2=5

(i) $\Delta = E - 1$ *(ii)* $\nabla = 1 - E^{-1}$

where E, Δ, ∇ are shift, forward and backward operators respectively.

(c) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of Regula-Falsi, correct to three decimal places. 5

4. *(a)* Using Secant method, find a real root of $e^x - x = 2$, correct to three decimal places. 4

(b) Using Gauss-elimination method solve the following equations :

$$2x + 2y + z = 12$$
$$3x + 2y + 2z = 8$$
$$5x + 10y - 8z = 10$$

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(c) Solve the following equations by Gauss-Seidel iteration method, correct to four decimal places :

$$7x_1 + 52x_2 + 13x_3 = 104$$
$$8x_1 + 11x_2 - 4x_3 = 95$$
$$3x_1 + 8x_2 + 29x_3 = 71$$

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5. *(a)* Evaluate $f(3{\cdot}8)$ from the following data :

x	:	0	1	2	3	4
$f(x)$	:	1	1·5	2·2	3·1	4·3

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(b) Using Modified Euler method, find y at $x = 0{\cdot}1$ and $x = 0{\cdot}2$ given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ with correct result upto four places of decimal. 4+4=8

(c) Using Iteration method, find a real root of $3x - log(x) - 16 = 0$, correct to fourth decimal places. 5

6. (a) Using Regula-Falsi method, find the real root of $xe^x - 2x + 1 = 0$, correct to three decimal places. 5

(b) Find $f'(1{\cdot}5)$ from the following tabulated function :

x :	1·5	2	2·5	3	3·5	4
$f(x)$:	3·375	7·0	13·625	24	38·875	59·0

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(c) Solve $\frac{dy}{dx} = \frac{1}{2}(1+x)y^2$ with $y(0) = 1$ at $x = 0{\cdot}2, 0{\cdot}4, 0{\cdot}6$ by Euler method and hence find $y(0{\cdot}8)$ by Milne's method. 8