53 (MA 401) NMCP

2019

NUMERICAL METHODS AND COMPUTER PROGRAMMING

Paper : MA 401

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Write an algorithm to implement Euler's method. Solve the differential equation $\frac{d^2y}{dx^2}=x^2+y^2$ with $y(0)=1$ using Euler's method by choosing a step size of $h=0{\cdot}2$ to obtain solution for $0 \le x \le 1$. 4+8=12

(b) Using Gauss-Seidel method solve the following equations : 8

$$2x+y+6z=9$$
$$8x+3y+2z=13$$
$$x+5y+z=7$$

Contd.

2. *(a)* Solve $e^x - 4x - 1 = 0$, correct to 4 places of decimal using the method of successive approximation. 5

(b) Fit a parabola $y = a + bx + cx^2$ to the following data : 10

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

(c) Prove that $\frac{E-1}{E} = \Delta$, where E is shift operator and Δ is forward difference operator. 5

3. *(a)* Using Newton's backward interpolation formula evaluate $f(3{\cdot}8)$ from the following data : 8

x :	0	1	2	3	4
$f(x)$:	1	1·5	2·2	3·1	4·3

(b) Using Gauss-elimination method solve the following equations : 7

$$2x + 2y + z = 12$$
$$3x + 2y + 2z = 8$$
$$5x + 10y - 8z = 10$$

(c) Using Newton-Raphson method, find a real root of $x^2 - x = 10$, correct to 4 decimal places. 5

4. *(a)* State Newton's forward interpolation formula. Compute $f(1{\cdot}2)$ from the following data : 8

x :	1	1·5	2	2·5	3
$f(x)$:	9	32·75	79	155·25	269

(b) Using Runge-Kutta method of fourth order, solve :

$$\frac{dy}{dx} = xy + y^2$$

with $y(0) = 1$ at $x = 0{\cdot}1, 0{\cdot}2, 0{\cdot}3$. 12

5. *(a)* Using modified Euler method, find y at $x = 0{\cdot}1$ and $x = 0{\cdot}2$, given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$ with correct result upto four places of decimal. 8

(b) Using Bisection method, find a real root of the equation $x^3 - 4x - 9 = 0$, correct to 3 decimal places. 6

(c) Use Trapezoidal rule with $n=4$ to estimate $\int_1^2 x^2\,dx$ and compare this estimation with exact value. 6

6. (a) Using Regula-Falsi method, solve $x^3-5x+2=0$, correct to 3 decimal places. 5

(b) Solve $\frac{dy}{dx}=\frac{1}{2}(x+y)y^2$ with $y(0)=1$ at $x=0{\cdot}2,\ 0{\cdot}4,\ 0{\cdot}6$ by Euler method and hence find $y(0{\cdot}8)$ by Milne's method. 8

(c) Compute first derivative of the function $f(x)$ at the point 0·1 from the following data : 7

x	:	0	1	2	3	4
$f(x)$	:	1	0	1	10	33