53 (MA 401) NMCP

2018

NUMERICAL METHODS AND C PROGRAMMING

Paper : MA 401

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Write an algorithm to implement Euler's Improved method. Using Euler's Improved method, find y at $x = 0{\cdot}1$ and $x = 0{\cdot}2$, given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$. 4+8=12

(b) Solve $x^3 + 4\sqrt{x} - 3 = 0$ correct to 3 places of decimal by Bisection Method. 8

2. (a) Using Milne's method, find $y(2)$ if $y(x)$ is the solution of $\frac{dy}{dx} = \frac{1}{2}(x+y)$, given that $y(0) = 2$, $y(0{\cdot}5) = 2{\cdot}636$, $y(1) = 3{\cdot}595$, $y(1{\cdot}5) = 4{\cdot}968$. 10

(b) Compute $f'(0{\cdot}1)$ from the following table : 6

x	:	0	1	2	3	4
$f(x)$	:	1	0	1	10	33

(c) What do you mean by relative error? Find the absolute error and the relative error if the number $X = 0{\cdot}004997$ is rounded off to three decimal places. 4

3. *(a)* The pressure and the volume of a gas are related by the equation $PV^{\gamma} = K$; γ and K being constants. Fit the equation to the following set of observations :

$P(kg/cm^2)$:	0·5	1·0	1·5	2·0	2·5	3·0
V (litres) :	1·62	1·00	0·75	0·62	0·52	0·46

10

(b) Using three point Gaussian quadrature formula evaluate $\int_0^1 \frac{dx}{1+x}$.

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(c) Using Regula-Falsi method, find the real root of $xe^x - 2x + 1 = 0$ correct to three decimal places. 5

4. *(a)* *(i)* Evaluate : $\frac{\Delta^2}{E} e^x . \frac{Ee^x}{\Delta^2 e^x}$, where h is the interval of difference.

(ii) If the interval of difference is unity, prove that :

$$y_4 = y_3 + \Delta y_2 + \Delta^2 y_1 + \Delta^3 y_0 .$$

3+3=6

(b) For a given function $f(x) = x^3$ and $\delta x = 0{\cdot}2$, taking five decimal places obtain the absolute errors in $f'(x)$ and $f''(x)$ at $x = 3$. 5

(c) Solve the following equations by Gauss-Seidel iteration method correct to four decimal places :

$$7x_1 + 52x_2 + 13x_3 = 104;$$

$$8x_1 + 11x_2 - 4x_3 = 95;$$

$$3x_1 + 8x_2 + 29x_3 = 71.$$ 9

5. *(a)* Using Newton's Interpolation formula, evaluate $f(3{\cdot}8)$ from the following data :

x	:	0	1	2	3	4
$f(x)$	:	1	1·5	2·2	3·1	4·3

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(b) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = xy + y^2$ with $y(0) = 1$ at $x = 0{\cdot}1, 0{\cdot}2, 0{\cdot}3$. 10

(c) Using Newton-Raphson's method, find a real root of $e^x = 4\sin x$ correct to three decimal places. 4

6. *(a)* The area A of a circle of diameter 'd' is given by the following :

d	:	80	85	90	95	100
A	:	5026	5674	6362	7088	7854

Find the approximate area of the circle of diameter 82. 5

(b) State and prove Newton's Forward Interpolation Formula. 7

(c) Solve the differential equation $\frac{dy}{dx} - xy = 0;\ \ y(0) = 1$ from $x = 0$ to $x = 0{\cdot}25$ using Euler's method. 8

7. (a) Integrate $\int_1^2 \frac{dx}{x}$ by calculus and by Simpson's formula taking eight divisions and compute $\log 2$. 6

(b) Using Gauss elimination method, solve the equations :

$$3x_1 + 2x_2 - 2x_3 = -2$$
$$2x_1 + 4x_2 + x_3 = 3$$
$$x_1 - x_2 + x_3 = 6$$ 7

(c) By means of Newton's divided difference formula find the value of $f(8)$ from the following table :

x	:	4	5	7	10	11	13
$f(x)$	:	48	100	294	900	1210	2028

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