2014

NUMERICAL METHODS & C PROGRAMMING

Paper : MA 401

Full Marks : 100

Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any** 5 (**five**) questions.*

1. (a) Write an algorithm to implement Euler's method. Solve the differential equation $\dfrac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$ using Euler's method by choosing an appropriate step size to obtain solution for $0 \le x \le 1$.   4+8=12

(b) The following are the number of deaths in four successive ten year age groups. Using Newton's formula for interpolation, find the number of deaths at 45-50 and 50-55 : 8

Age Group :	25-35	35-45	45-55	55-65
Deaths :	13229	18139	24225	31496

2. (a) Find a real root of the equation $x^3 - 6x + 4 = 0$ by using Newton-Raphson method correct to three places of decimals. 8

(b) Fit a curve of the form $y = ab^x$ to the following data :

x :	2	3	4	5	6
y :	8·3	15·4	33·1	65·2	127·4

8

(c) If $x = 2 \cdot 536$, find the absolute error and relative error when :

(i) x is rounded off

(ii) x is truncated to two decimal places. 4

3. (a) Using modified Euler's method, find y at $x = 1 \cdot 2$ and $x = 1 \cdot 4$ given that

$$\frac{dy}{dx} = \frac{2y}{x} + x^3 \; ; \; y(1) = 0 \cdot 5.$$ 8

(b) Given the following, find $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$ at $1 \cdot 1$ and $1 \cdot 6$:

x :	1	1·1	1·2	1·3	1·4	1·5	1·6
y :	7·989	8·403	8·781	9·129	9·451	9·750	10·031

8

(c) If $u = 4x^6 + 3x - 9$, find the relative percentage of error in computing $x = 1 \cdot 1$ given that the error in x is $0 \cdot 05 \%$. 4

4. (a) Find a positive root of the equation $x^4 - x - 10 = 0$ correct to three decimal places using Secant Method. 7

(b) Apply Gauss-elimination method to solve the following :

$$5x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 7x_2 + x_3 + x_4 = 12$$

$$x_1 + x_2 + 6x_3 + x_4 = -5$$

$$x_1 + x_2 + x_3 + 4x_4 = -6 \qquad 8$$

(c) Use Simpson's 1/3 rd rule to find $\displaystyle\int_0^{0.6} e^{-x^2}\, dx$

by taking seven ordinates. 5

5. *(a)* Using Runge-Kutta method of fourth order,

solve $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at

$x = 0 \cdot 2, 0 \cdot 4.$ 8

(b) State Newton's divided difference formula. Find the polynomial of the lowest possible degree which assumes the values 1245, 33, 5, 9 and 1335 at $x = -4, -1, 0, 2$ and 5 respectively. Also find the value of the polynomial at the abscissa 1. 2+4+2=8

(c) Explain graphically how to obtain Regula-Falsi Formula. 4

6. (a) Apply Gauss-Seidal iteration method to solve :

$$28x + 4y - z = 32$$

$$x + 3y + 10z = 24$$

$$2x + 17y + 4z = 35 \qquad 7$$

(b) Use Lagrange's interpolation formula, find the form of the function $f(x)$ and hence find the value of $f(3)$, from the following data : 6

x	:	0	1	2	5
$f(x)$	:	2	3	12	147

(c) Find the real root of the equation $e^x - 3x = 0$ by Bisection method correct to four decimal places. 7

7. *(a)* Solve $\dfrac{dy}{dx} = \dfrac{1}{2}(1+x)\,y^2$ with $y(0)=1$ at $x = 0\cdot2,\ 0\cdot4, 0\cdot6$ by an appropriate method and hence find $y(0\cdot8)$ by Milne's method.   10

(b) Find a real root of $cos\,x = 3x+1$ by the method of successive approximations correct to 3 decimal places.   5

(c) Evaluate $\displaystyle\int_0^2 \dfrac{x^2+2x+1}{1+(x+1)^4}\,dx$ by Gaussian 3-point formula.   5