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53 (MA 302) DSMA-III

2017

DISCRETE MATHEMATICS

Paper : MA-302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* If *A*, *B* and *C* are any three sets, then show that

$$(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$$ 4

(b) Let $f:R\to R$ defined by $f(x)=x^3-x$. Show that *f* defines a mapping from *R* onto *R*. Is this a one-one mapping ? Justify. 2+1=3

(c) If $axa=b$ in a group *G*, then find the value of *x*. 3

(d) Define subgroup. Show that a non-empty subset H of a group G to be a subgroup of G if and only if $a \in H,\ b \in H \Rightarrow ab^{-1} \in H$. 2+8=10

2. *(a)* If P_n is the symmetric group of n symbols, then show that the alternating group A_n is a normal subgroup of P_n. 6

(b) Prove that if for every element 'a' in a group G such that $a^2 = e$, where e is the identity in G, then G is abelian. 4

(c) Show that the sum of degrees of all the vertices in a graph G is even. 4

(d) Let $(P(A), \leq)$ denote the standard partial order on the subsets of A, given by set inclusion. Then

(i) Determine whether there is a greatest and least element of $(P(A), \subseteq)$.

(ii) Draw the Hasse diagram of $(P(\{a,b,c\}), \subseteq)$ 4+2=6

3. *(a)* Prove that the statement $(p \to q) \to (p \wedge q)$ is a contingency. 4

(b) Prove that a finite graph is bipartite if and only if it contains no cycles of odd length. 6

(c) Prove that every chain is a lattice. 3

(d) Prove that for a connected graph, any two longest path have a common point. 7

4. *(a)* If f is a homomorphism of a group G into a group G', then show that

(a) $f(e) = e'$, where e is the identity of G and e' is the identity of G'.

(b) $f(a^{-1}) = (f(a))^{-1}, \forall a \in G.$

3+3=6

(b) Let $(L, \leq)$ be any lattice and a, x, y be elements of L such that $x \leq y$. Show that $a \wedge x \leq a \wedge y$. 5

(c) Consider a set $S = \{1, 2, 3\}$. Is the relation of set inclusion "$\subseteq$" in a partial order on $P(S)$? Where $P(S)$ is a power set of S. 4

(d) Show that intersection of any two subgroups of a group G is a subgroup of G. 5

5. (a) Let I be the set of all integers. Define a relation R on I by xR_y if and only if $x-y$ is divisible by 3, $\forall x, y \in I$. Show that R is an equivalence relation on I. 6

(b) Express the following permutations as the product of disjoint cycles.

(i) (1 2 3) (4 5) (1 6 7 8 9) (1 5)

(ii) (1 3 2 5) (1 4 3) (2 5 1)

4+3=7

(c) Negate each of the following propositions :

(i) All boys can run faster than girls.

(ii) Some students do not live in hostel.

(iii) Some of the students are absent and the classroom is empty.

1+1+1=3

(d) Show that every simple graph (finite) has two vertices of the same degree. 4

6. *(a)* Prove that for any propositions p, q, r, the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. 3

(b) Find the principal conjunctive normal form of the compound statement $p \leftrightarrow (\sim pV \sim q)$. 3

(c) Let $A(x)$: x has a white colour

$B(x)$: x is a polar bear

$C(x)$: x is found in cold region, over the universe of animals.

Translate the following into simple sentences :— 1+1+1=3

(i) $\exists x\,(B(x) \wedge \vee A(x))$

(ii) $(\exists x)(\sim c(x))$

(iii) $(\forall x)(B(x) \wedge c(x) \to A(x))$

(d) Draw a graph whose every edge is a bridge. 2

(e) Define 3-regular graph. Draw a 3-regular graph of five vertices. 1+2=3

(f) Define cyclic group. Give an example of an infinite cyclic group. 1+1=2

(g) How many generators are there in a cyclic group of order 8 ? Also, write the generators with justification. 1+3=4
