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53 (MA 302) DSMA

2019

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Define subgroup. Show that a non-empty subset H of a group G to be a subgroup of G if and only if $a \in H, b \in H \Rightarrow ab^{-1} \in H$. 2+8=10

   (b) Express the following permutations as the product of disjoint cycles

   (i) (1 2 3) (4 5) (1 6 7 8 9) (1 5)

   (ii) (1 3 2 5) (1 4 3) (2 5 1) 4+3=7

(c) Define cyclic group. Give an example of an infinite cyclic group. 2+1=3

2\. (a) Define adjacent matrix for undirected graph. Draw the undirected graph represented by adjacency matrix A given by

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

2+3=5

(b) Prove that for a connected graph, any longest path have a common point. 7

(c) Let $(L, \leq)$ be any lattice and a, x, y be elements of L such that $x \leq y$. Show that $a \wedge x \leq a \wedge y$. 5

(d) Find the principal conjunctive normal form of the compound statement $p \leftrightarrow (\sim p \vee \sim q)$. 3

3\. (a) If A, B and C are any three sets, then show that

$$(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$$

4

(b) If f is a homomorphism of a group G into a group G', then show that

(a) $f(e) = e'$, where e and e' are the identities of G and G' respectively.

(b) $f(a^{-1}) = (f(a))^{-1}, \ \forall a \in G$

3+3=6

(c) Prove the following using symbolic logic :

If prices are high, then wages are high. Prices are high or there are price controls. Also, if there are price controls then there is not an inflation. However, there is an inflation. Therefore wages are high. 5

(d) Consider the set $\mathbb{N} \times \mathbb{N}$, the set of ordered pairs of natural numbers. Let R be a relation on $\mathbb{N} \times \mathbb{N}$ which is defined by $(a, b)^R (c, d)$ if and only if $a + d = b + c$. Prove that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. 5

4. *(a)* Consider the set $S=\{a,b,c\}$. Prove that $(P(S),\subseteq)$ is a poset. Draw the Hasse diagram of the poset $(P(S),\subseteq)$. 4+2=6

(b) Show that a lattice is a partially ordered set. 4

(c) Find the order of each element of the multiplicative group $G=\{1,\omega,\omega^2\}$. 3

(d) Solve the recurrence relation

$$a_{n+2}-2a_{n+1}+a_n=2^n,\ a_0=2, a_1=1$$

by the method of generating function. 7

5. *(a)* Prove that the mapping $f:A\to B$ in one-one onto if and only if there exists a mapping $g:B\to A$ such that $g\circ f$ and $f\circ g$ are the identity mapping on A and B respectively. 5

(b) Translate the following sentences into logical expressions :

(i) Some students are not successful.

(ii) Any integer is either positive or negative.

(iii) Every person is precious.

(iv) No student of this class is perfect in calculation. 4

(c) Establish the equivalence

$$p\to q\vee r\equiv p\wedge(\sim p)\to r\equiv p\wedge(\sim r)\to q$$

Hence write the following sentences in two different ways. If n is prime, then n is odd or n is 2. 3+2=5

(d) Show that for any graph with six points G or $\overline{G}$ contains a triangle. 6

6. *(a)* Negate each of the following propositions :

(i) All boys can run faster than girls.

(ii) Some students do not live in hostel.

(iii) Some of the students are absent and the classroom is empty. 1+1+1=3

(b) Draw a 3-regular graph of five vertices. 2

(c) Prove that the statement $(p \to q) \to (p \wedge q)$ is a contingency. 5

(d) If P_n is the symmetric group of n symbols, then show that the alternating group A_n is a normal subgroup of P_n. 6

(e) Find the negation of each of the following quantified predicates :

(i) $(\exists\, x \in D)(x + 2 = 7)$

(ii) $(\forall x \in D)(x + 3 < 10)$,

where $D = \{1, 2, 3, 4\}$.

2+2=4