Total number of printed pages–11

53 (MA 302) DSMA

2018

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* *(i)* Prove that, for any two sets A and B, $(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$ 4

(ii) If $f : X \to Y$ be one-one and onto map, then prove that $f\circ f^{-1} = I_Y$ and $f^{-}\circ f = I_X$, where I_Y and I_X are the identity mappings on Y and X respectively. 5

Contd.

(b) Show that

$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology. 3

(c) Simplify the following Boolean function using K-map. 5

$$f = x'_1 x_2 x'_3 x'_4 + x'_1 x_2 x'_3 x_4 + x'_1 x'_2 x_3 x_4 + x'_1 x'_2 x_3 x'_4 + x_1 x_2 x_3 x_4 + x_1 x_2 x_3 x'_4 + x_1 x'_2 x'_3 x'_4 + x_1 x'_2 x'_3 x_4 + x_1 x_2 x'_3 x'_4 + x_1 x_2 x'_3 x_4$$

(d) (i) Distinguish between simple graph and multigraph with an example each. 2

(ii) Let G be a regular graph of degree 3 with 6 vertices. Find the number of edges of the graph. 1

2. (a) (i) Show that the set II of all integers is a ring with respect to addition and multiplication of integers as the two ring compositions. 6

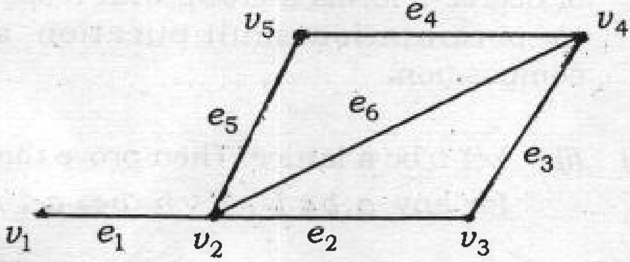
(ii) Show that every subgroup of an abelian group is normal. 3

(b) Express the following Boolean function in DNF : 3+1=4

$(x+y).(x+z)+y+z'$

Hence find its CNF.

(c) Let G be a graph defined as follows :



(i) Draw the graph $G-\{v_2\}$ and $G-\{e_4, e_6\}$.

(ii) Find a path of length 4 and a trail of length 3.

(iii) Write all its cycles.

2+2+1=5

(d) Let $Ax : x$ is an animal.

Mx : x is mortal.

Then write the following in sentence :

(i) $(\forall x)(Ax \to Mx)$

(ii) $(\exists x)(Ax \wedge Mx)$

1+1=2

3. (a) Show that the set P_3 of all permutations of degree 3 forms a group with respect to permutation multiplication as composition. 5

(b) (i) Let L be a lattice. Then prove that, for any $a, b \in L$, $a \vee b = b \Leftrightarrow a \leq b$. 3

(ii) Let $S = \{a, b, c\}$ and $X = P(S)$. Define a relation '$\leq$' on X such that $A \leq B$ if and only if $A \subseteq B, \forall A, B \in X$. Show that $(X, \leq)$ is a poset. 3

(c) (i) Construct the truth table of the following :

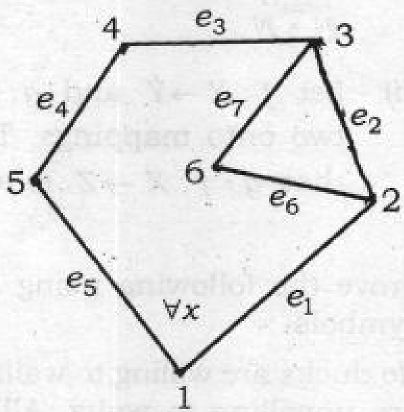
$[p \to (q \to r)] \to [(p \wedge q) \to r]$ 3

(ii) Find the scopes of '$\exists x$' and '$\forall x$' of the following statement :

$$(\exists x)(\forall y)(xy=0) \wedge (\forall x)(\forall z)(x+z=z+x)$$

1+1=2

(d) Consider the following graph G :



Let $X_i = \{i\} \cup \{x \mid x$ is an edge incident with the vertex $i\}$, $\forall i = 1, 2, ..., 6$. Then construct a graph G' with $X_i, \forall i = 1, 2, ..., 6$ as its vertices such that there is an edge between X_i and X_j, $\forall i = 1, 2, ..., 6$ as its vertices such that there is an edge between X_i and if and only if $X_i \cap X_j \neq \phi$.

4

4. *(a)* *(i)* Let R be a relation defined on $\mathbb{N} \times \mathbb{N}$ by

$$(a, b) R (c, d) \Leftrightarrow ad = bc, \quad a, b, c, d \in \mathbb{N}$$

and $\mathbb{N}$ is the set of all natural numbers. Then show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. 5

(ii) Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two onto mappings. Then prove that $g \circ f : X \rightarrow Z$ is also onto. 2

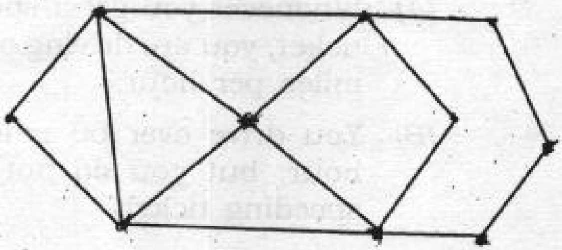
(b) Prove the following using logical symbols :

No ducks are willing to waltz. No officers are unwilling to waltz. All my poultry are ducks. Therefore, none of my poultry are officers. 5

(c) *(i)* Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, where

$V_1 = \{u_1, u_2\}, V_2 = \{v_1, v_2, v_3\}, E_1 = \{\{u_1, u_2\}\}$ and $E_2 = \{\{v_1, v_2\}, \{v_2, v_3\}\}$, be two graphs. Then draw the graph of $G_1[G_2]$. 2

(ii) Examine whether the following graph is Eulerian or not. If so, find an Eulerian circuit of it. Is the graph Hamiltonian ? Give justification. 1+1+1=3



(d) Draw a logic diagram to represent the following Boolean function :

$$[(x_1 + x_2).(x'_1+x_3)]+[(x_3 + x_4)'.(x'_2+x_3)']$$

3

5. (a) (i) Prove that a non-empty subset H of a group G is a subgroup if and only if $HH^{-1} = H$. 6

(ii) Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$. 5

(b) (i) Let p : You drive over 60 miles per hour.

q : You get a speeding ticket.

Then express the sentences given below in symbolic language :

(A) Whenever you get a speeding ticket, you are driving over 60 miles per hour.

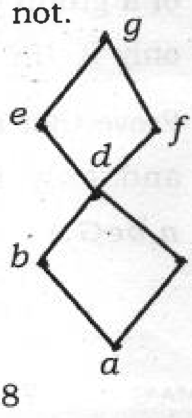
(B) You drive over 60 miles per hour, but you do not get a speeding ticket. 1+1=2

(ii) Express the following sentences in symbolic form : 1+1=2

(A) No freshmen are dignified.

(B) Some freshmen are pretty.

(c) (i) Examine whether the following Hasse diagram defined on the set $A=\{a, b, c, d, e, f, g\}$ represents a lattice or not. 3



(ii) For any lattice L, prove that

$$(a+b)' = a'.b' \ \forall a, b \in L$$ 2

6. (a) If f is a homomorphism of a group G into a group G', then show that

(i) $f(e) = e'$, where e and e' are the identities of G and G' respectively.

(ii) $f(a^{-1}) = [f(a)]^{-1}, \forall a \in G.$

3+2=5

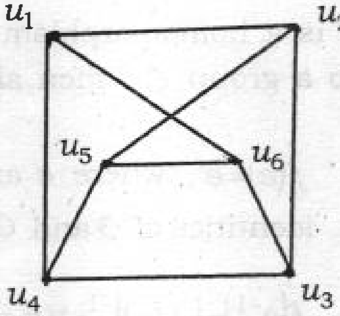
(b) (i) Prove the following using logical symbols :

If A wins, then either B or C will place. If B places, then A will not win. If D places, then C will not. Therefore, if A wins, D will not place. 5

(ii) Find the negative of the following expression :

$$(\exists x)(Ax \wedge Bx).$$ 1

(c) (i) Draw the bipartite graph of the following graph :



Is it a complete bipartite graph ? Give justification. 2+1=3

(ii) State true **or** false of the following statements :

(A) A graph G of order p is a tree, if it is a cyclic and has size p-1

(B) Every tree of order two or more has at least two terminal vertices.

1+1=2

(d) Let $A=\{1,2,3,4,12\}$. Consider the partial order relation "$\leq$" on A such that $a \leq b$ if and only if $a|b$, $a,b \in A$. Then draw the Hasse diagram of the poset $(A,\leq)$ showing the diagraph of it. Find also the least and the greatest element of the poset. 3+1=4
