2014

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

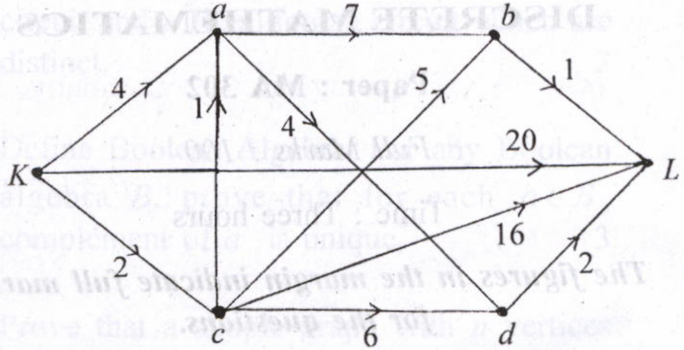
1. *(a)* Show that every simple finite graph has two vertices of the same degree. 4

   (b) Using Big-Oh notation estimate the growth of 3

   $$f(n) = 6n^2 + 5n + 7\log(n!)$$

   (c) Define a field. Give an example of a field with five elements. 5

(d) Find the shortest path between K and L in the graph shown below by using Dijkstra's Algorithm. 8



2. (a) What is a cyclic group ? Give an example of a cyclic group of order 4. What are the generators of this group ? Justify your answers. 2+1+2=5

(b) Show that the sequence $a_n = 3(-1)^n + 2^n - n + 2$ is a solution of the recurrence relation 3

$$a_n = a_{n-1} + 2a_{n-2} + 2n - 9$$

(c) If $(A, \leq_1)$ and $(B, \leq_2)$ are posets, then prove that $(A \times B, \leq)$ is a poset with partial order '$\leq$' defined by $(a,b) \leq (a',b')$ if $a \leq_1 a'$ in A and $b \leq_2 b'$ in B. 5

(d) Show that every chain is a distributive lattice. 4

(e) Let $R = \{(1,1),(1,2),(2,3),(2,4),(3,4),(4,1),(4,2)\}$ and $S = \{(3,1),(4,4)(2,3),(2,4),(1,1),(1,4)\}$. Are SoR and RoS equal ? Justify. 3

3. (a) Prove that in any graph, there are an even number of vertices of odd degree. 4

(b) Using truth table test the validity of following argument

"If I will select in I.A.S examination, then I will not be able to go to London. Since, I am going to London, I will not select in I.A.S Examination." 3

(c) Find the number of solutions to each equation, where the variables are non-negative integers 2

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10$$

(d) (i) What is a ring ? Give an example of a finite ring and a non-commutative ring. 6

(ii) Let $P = \{3.a, 2.b, 2.c\}$,
$Q = \{5.a, 4.b, 1.d\}$

then find the cardinalities of

$P \cup Q,\ P \cap Q; P - Q; Q - P; P + Q$ 5

4. (a) There are 1400 students in a college ; 1250 can play basketball, 952 can play cricket and 60 students can neither play basketball nor cricket. How many students can play both basketball and cricket ? 3

(b) Show that the relation R on the set of Z defined by "aRb iff $a-b$ is divisible by 5" is an equivalence relation, where Z is the set of integers. 5

(c) Prove that a complete graph with n vertices contains $\frac{n(n-1)}{2}$ edges. 4

(d) (i) Let $A(x)$: x has a white colour 3

$B(x): x$ is a polar bear

$C(x): x$ is found in cold region over the universe of animals.

Then translate the following into simple sentences.

(p) $\exists x\,(B(x) \wedge \sim A(x))$

(q) $(\exists x)(\sim C(x))$

(r) $(\forall x)(B(x) \wedge C(x) \longrightarrow A(x))$

(ii) Find the principal conjunctive normal form using truth table of $(p \wedge q) \vee (\sim q \wedge r)$. 5

5. (a) (i) Show that the map $f: R \rightarrow R$ defined by $f(x) = 5x - 3$ is a bijective map. 2

(ii) State true *or* false with justification $(\{1,-1\},t)$ is a finite group of order 2 2

(b) Is a non-symmetric relation an anti-symmetric relation ? Justify your answer by citing examples to show the difference between them. 4

(c) Prove that the vertex connectivity of a graph G is always less than or equal to the edge connectivity of G. 5

(d) How many vertices do the following graphs have if they contain 4

(i) 16 edges and all vertices of degree 2

(ii) 21 edges, 3 vertices of degree 4 and others each of degree 3.

(e) Write the negation of each of the following statements : 3

(i) Paris is in India and Delhi is in France.

(ii) $2+4=6$ and $12>7$

(iii) He swims if and only if the water is warm.

6. (a) Construct the truth table for the compound proposition

$$(\sim p \rightarrow r) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$ 3

(b) Examine the function $g(x) = x^2 - 1$ on the set of real numbers for one-one and onto. 3

(c) Does the multiplicative inverse of $\begin{pmatrix} 4 & 5 \\ 1 & 3 \end{pmatrix}$ of the ring of all matrices of order two over the integers exists ? Justify. 3

(d) How many numbers each lying between 100 and 1000 can be formed with the digits 2, 3, 4, 0, 8, 9 ? 3

(e) If L be a lattice, then for every a, b, in L, show that 8

(i) $a \wedge b = a$ if and only if $a \leq b$

(ii) $a \vee (a \wedge b) = a$

(iii) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$ $\forall a, b, c \in L$

(with usual meaning of the symbols)

7. (a) Give an example of a many-to-one function that is also onto. 2

(b) State *true* **or** *false* with justification that $a-b$ in the set of natural numbers is a binary operation. 2

(c) Give an example of a graph that has an Euler circuit and a Hamiltonian circuit which are distinct. 2

(d) Define Boolean Algebra. For any Boolean algebra B, prove that for each $a \in B$, complement of a is unique. 3

(e) Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. 5

(f) (i) Obtain an equivalent expression for $[(x.y)(z'+xy')]$ 3+3=6

(ii) Find Karnaugh maps and simplify the expressions

$AB'+A'B$ and $AB'+A'B'$

(with usual meaning of the symbols)
