2013

(May)

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Let $f : X \to X$ and $g : Y \to X$. Then g is the inverse function of f, if the product function $(g.f) : X \to X$ is identity function on X and $(f.g) : Y \to Y$ is the identity function on Y. 6

   (b) Show that the necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that $a \in H$, $b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G. 8

Contd.

(c) Prove that a lattice L is complete if and only if it has its zero and unity in it. 6

2. (a) Write the axioms of a Boolean algebra $(B, +, \bullet, /)$. Show that for all $a, b, c \in B$

   (i) if $b+a=c+a$ and $b+a'=c+a'$, then $b=c$

   (ii) $a+b'=1$ if and only if $a+b=a$.

   2+2+2=6

(b) Prove that a graph G is a tree if and only if there is a unique path between *any two* vertices. 6

(c) Construct a truth table for the statement formula : 4

$$(p \wedge \sim q) \vee (q \wedge (\sim p \vee r))$$

(d) Show that if a, b are any two elements of a group G, then $(ab)^2 = a^2 b^2$ if and only if G is abelian. 4

3. (a) Let I be the set of all integers and define a relation R in I such that xRy holds if and only if $x-y$ is divisible by 5. Show that R is an equivalence relation. 5

(b) Show that the product of two elements of a group G is the product of the inverses taken in the reverse order. 4

(c) Prove that for any connected graph G having n vertices and $(n-1)$ edges is a tree. 4

(d) Prove that the order of each subgroup of a finite group is a divisor of the order of the group. 7

4. (a) Let L be a distributive lattice. Show that for any $a, b, c \in L$,

$$(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a)$$

5

(b) Find the conjunctive normal form and disjunctive normal forms of the formulae $(q \to p) \wedge (\sim p \wedge q)$ (using truth table).

2½+2½=5

(c) Translate each of the following sentences into a statement formula : 4×1=4

(i) A necessary condition for x to be prime is that either x is odd or $x = 2$.

(ii) Taxes will be raised only if the budget is not cut.

(iii) He will come today unless it rains.

(iv) He is poor but honest.

(d) Show that every group of prime order is cyclic. Is the converse true ? Justify your answer. 5+1=6

5. (a) If A and B are any two sets, prove that

(i) $A-(B\cup C)=(A-B)\cap(A-C)$

(ii) $A\cap(B-C)=(A\cap B)-(A\cap C)$

3+3=6

(b) Draw a circuit using only NAND-gates that represents the Boolean functions : 2+2=4

(i) $f(x, y)=x+y$

(ii) $f(x, y)=xy$

(c) Let $(L, \wedge, \vee)$ be a lattice with the induced partial order $\leq$. Show that for $a, b, c\in L$

(i) $c\leq a$ and $c\leq b\Rightarrow c\leq a\wedge b$

(ii) $a\leq c$ and $b\leq c\Rightarrow a\vee b\leq c$ 2+2=4

(d) Find the negation of each of the following quantified predicates : 2+2=4

(i) $(\exists\, x\in D)\,(x+2=7)$

(ii) $(\forall\, x\in D)\,(x+3<10)$, where $D=\{1, 2, 3, 4\}$

(e) Consider the set $S=\{a, b, c\}$ and consider the poset $(p(s), \subseteq)$, Draw the Hasse-diagram of that poset. 2

6. (a) Prove that a simple graph with at least two vertices has same degree. Also if the simple graph G has v vertices and e edges, then how many does G' (complement of G) have ? 4+2=6

(b) If R is a ring such that $a^2 = a, \quad \forall a \in R$, prove that

(i) $a+a=0, \ \forall a \in R$

(ii) $a+b=0, \Rightarrow a=b, \ \forall a, b \in R$

(iii) R is a commutative ring. 2+2+2=6

(c) Express the Boolean expression $(x+y)(x+y')(x'+z)$ in DNF in the variables x, z and also express it in DNF in the variables x, y, z. 3+3=6

(d) If $p \rightarrow q$ is F, determine the truth value of $(\sim(p \wedge q)) \rightarrow q$. 2

7. *(a)* Let $f : U \to V$, $g : V \to X$ and $h : X \to Y$ be any three mappings, then show that 2+1+2=5

(i) $h.(g.f) = (h.g).f$

(ii) If f and g are both one-to-one then so is $g.f$.

(iii) If f and g are both onto, then so is $g.f$.

(b) If H in a subgroup of a group G such that index of H in G is 2, then prove that H is a normal subgroup of G. 5

(c) Show that the order of an element of a group G is the same as that of its inverse. 5

(d) Show that a lattice is a partially ordered set. 5