53 (MA 302) DSMA

2013

(December)

DISCRETE MATHEMATICS

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* What is a cyclic group ? Give an example of a cyclic group of order 4. What are the generators of this group ? Justify your answers. 5

   (b) Show that the relation R on the set of integers Z defined by "aRb if and only if $a-b$ is a multiple of 3" is an equivalence relation. 5

   (c) A simple graph with n vertices and k-components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. 4

(d) Negate the following statements 2+2+2=6

(i) For all real numbers x, if $x>3$ then $x^2>9$

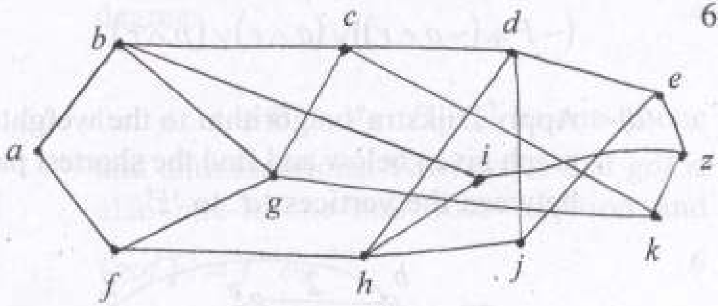
(ii) All students in the class have taken a course of discrete mathematics.

(iii) This computer program is correct if and only if, it produces the correct answer for all possible sets of input data.

2. (a) Let P and Q be the multisets where $P=\{5.a, 7.b, 8.c\}$ and $Q=\{3.a, 5.b, 9.c\}$. Find the cardinalities of $P \cup Q$, $P \cap Q$, and $P \setminus Q$. 6

(b) Define field. Give an example of a field with finite elements. 3

(c) Find the shortest path from vertex a to z and its length from the graph given below. 6



(d) Obtain the principal disjunctive normal form of $(P \wedge \sim q \wedge \sim r) \vee (q \wedge r)$.

Also find its disjunctive normal form. 3+2=5

3. (a) Give an example of 2+2=4

(i) a commutative ring without identity element.

(ii) a non-commutative ring with identity elements.

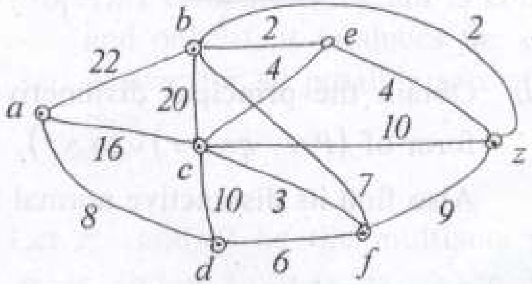
(b) Using the big-oh notation, estimate the growth of the function

$f(n) = 2n^3 - 3n^2 + 4n$ 2

(c) Find the truth table of the following compound proposition

$$(\sim P \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \qquad 4$$

(d) Apply Dijkstra's algorithm to the weighted graph given below and find the shortest path between the vertices 'a' to 'z'. 10



with usual meaning of the symbols.

4. (a) Define partially ordered sets. Examine whether the set Z^+ of all positive integers under divisibility relation forms a poset or not. 1+3=4

(b) Show that $G=\{a+b\sqrt{2} : a,b \in Q\}$ is a group under addition, where Q is the set of rational numbers. 6

(c) Prove that a simple graph with at least two vertices has at least two vertices of same degree. 4

(d) Let $f: A \to B$ and $g: B \to C$ be one-to-one and onto functions. Then prove that gof is also one-to-one and onto function and $(gof)^{-1} = f^{-1}og^{-1}$. 6

5. (a) Define adjacency matrix. Draw the undirected graph represented by adjacency matrix A given by 1+4=5

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

(b) Prove that if x^2 is divisible by 4, then x *is* even. 4

(c) What do you mean by a lattice ? Show that the power set $(P(S), \subseteq)$ is a lattice for $S \neq \phi$ 4

(d) (i) Show that the sequence $a_n = 3(-1)^n + 2^n - n + 2$ is a solution of the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2} + 2n - 9$$ 3

(ii) Find the multiplicative inverse of $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ of the ring of all matrices of order two over the integers. 4

6. (a) Define Hamiltonian and Eulerian graph. Give an example of a graph which is Hamiltonian but not Eulerian and vice-versa. 2+2=4

(b) A computer code word is to consist of two distinct alphabets followed by two distinct integers between 1 and 9. 2+2=4

(i) How many such code words are there ?

(ii) How many of them end with even integers ?

(c) Represent the argument symbolically 3

If it rains today, then we will not have a party today.
If we do not have party today then we will have a party tomorrow.

....

Therefore, if it rains today, then we will have a party tomorrow.

Determine whether the above argument is valid.

(d) Let $A=\{1,3,9,27,81\}$. Examine whether A is a chain under divisibility " / ". If so, draw the Hasse diagram. 3+1=4

(e) Show that G is an abelian group if and only if $(ab)^2 = a^2b^2 \ \forall a,b \in G$ 5

7. (a) Give an example (with justification) of the following 2×3=6

(i) a many-to-one function that is also onto.

(ii) a one-to-one function that is not onto.

(iii) an onto function that is not one-to-one.

(b) Translate the following sentences into logical expressions 3

(i) All men are mortal

(ii) There is a student who likes mathematics but not biology.

(iii) Some men are genius.

(c) Show that for any graph G with six points, G or $\overline{G}$ contains a triangle. 4

(d) Find the number of solutions to each equation, where the variables are non-negatives integers

$x_1 + x_2 + x_3 + x_4 = 10$ 2

(e) Prove that a non-empty subset H of a group G is a subgroup of G if only if

(i) $a \in H, b \in H \Rightarrow a * b \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$, where a^{-1} is the inverse of a in G. 5
