Total number of printed pages–6

53 (MA 302) DMAT

2021

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Show that for any graph with six points G or $\overline{G}$ contains a triangle. 7

   (b) Define adjacency matrix for undirected graph. Draw the undirected graph represented by adjacency matrix A given by : 2+3=5

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Contd.

(c) Obtain the principal disjunctive normal form of $(p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$. 4

(d) Translate the following sentences into logical expressions : 1×4=4

(i) Every person is precious.

(ii) Any integer is either positive or negative.

(iii) Some students are not successful.

(iv) All students in a class have not taken Discrete Mathematics.

2\. (a) If A, B, C are any three sets, then prove that : 3+2=5

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $(A \cup B)^c = A^c \cap B^c$

(b) Define group. Show that the set $G = \{a + b\sqrt{2} : a, b \in Q\}$ where Q is the set of rational numbers, is a group with respect to addition. 2+4=6

(c) Prove that the mapping $f : A \to B$ is one-one onto if and only if there exists a mapping $g : B \to A$ such $g \circ f$ and $f \circ g$ are the identity mapping on A and B respectively. 4

(d) Define a subgroup of a group. If H and K are subgroups of a group G, then show that HK is a subgroup of G if and only if $HK = KH$. 5

3. (a) Show that every cubic group has an even number of points. 5

(b) Show that the set of all positive integers under divisibility relation form a poset. 5

(c) Consider the set $\mathbb{N} \times \mathbb{N}$, the set of ordered pairs of natural numbers. Let R be a relation on $\mathbb{N} \times \mathbb{N}$ which is defined by $(a, b)\ R\ (c, d)$ if and only if $ad = bc$. Prove that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. 5

(d) Show that if a and b are any two elements of a group G, then $(ab)^2 = a^2 b^2$ if and only if G is abelian. 5

4. (a) Define normal subgroup. Show that a subgroup H of a group G is normal if and only if $xHx^{-1} = H\ \forall\ x \in G$. 1+4=5

(b) Write down all the permutations on four symbols 1, 2, 3, 4. Which of these permutations are even? 3+2=5

(c) Consider the set $S=\{a,b,c\}$. Prove that $(P(S),\subseteq)$ is a poset. Draw the Hasse diagram of the poset $(P(S),\subseteq)$. 4+2=6

(d) Show that a lattice is a partially ordered set. 4

5. (a) Is the relation "in brother of" an equivalence relation on a set of all human beings ? Why ? 3

(b) If G is a group, then prove that $(ab)^{-1}=b^{-1}a^{-1}\,\forall a,b\in G$. 3

(c) Let R_0 denote the set of all non-zero real numbers. Prove that the map $f: R_0 \to R_0$ defined by

$f(x)=\frac{1}{x} \quad \forall x \in R_0$ is bijective.

4

(d) Examine whether the following compound proposition in tautology or not.

$(p\to q)\leftrightarrow(\sim q\to\sim p)$ 2

(e) Find the negation of each of the following quantified predicates : 2+2=4

(i) $(\exists x D)(x+2=7)$

(ii) $(\forall x \in D)(x+3<10)$,
where $D=\{1, 2, 3, 4\}$.

(f) Show that in a non-directed graph, the total number of odd degree vertices is even. 4

6. (a) Establish the equivalence :

$$p \rightarrow q \vee r \equiv p \wedge (\sim p) \rightarrow r \equiv p \wedge (\sim r) \rightarrow q$$

Hence write the following sentence in two different ways. If n is prime, then n is odd or n is 2. 3+2=5

(b) If L is a lattice, then for every $a, b \in L$ prove that—

(i) $a \vee b = b$ iff $a \leq b$;

(ii) $a \wedge b = a$ iff $a \leq b$

with usual meaning for the symbols. 5

(c) Let A and B be two non-empty sets. If $f : A \to B$ is invertible, then show that f^{-1} is also invertible. 5

(d) Prove that every cyclic group is abelian, but the converse may not be true in general. Give an example to show that every abelian group is not cyclic. 5