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53 (MA 302) DMAT

2021

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer **any five** questions.

1. (a) Show that for any graph with six points G or \bar{G} contains a triangle. 7
- (b) Define adjacency matrix for undirected graph. Draw the undirected graph represented by adjacency matrix A given by: 2+3=5

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Contd.

(c) Obtain the principal disjunctive normal form of $(p \wedge \sim q \wedge \sim r) \vee (q \wedge r)$. 4

(d) Translate the following sentences into logical expressions: 1×4=4

(i) Every person is precious.

(ii) Any integer is either positive or negative.

(iii) Some students are not successful.

(iv) All students in a class have not taken Discrete Mathematics.

2. (a) If A, B, C are any three sets, then prove that: 3+2=5

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

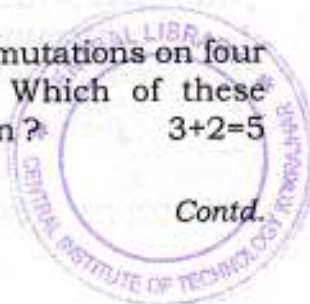
(ii) $(A \cup B)^c = A^c \cap B^c$

(b) Define group. Show that the set

$G = \{a + b\sqrt{2} : a, b \in Q\}$ where Q is the set of rational numbers, is a group with respect to addition. 2+4=6

(c) Prove that the mapping $f: A \rightarrow B$ is one-one onto if and only if there exists a mapping $g: B \rightarrow A$ such $g \circ f$ and $f \circ g$ are the identity mapping on A and B respectively. 4

- (d) Define a subgroup of a group. If H and K are subgroups of a group G , then show that HK is a subgroup of G if and only if $HK = KH$. 5
3. (a) Show that every cubic group has an even number of points. 5
- (b) Show that the set of all positive integers under divisibility relation form a poset. 5
- (c) Consider the set $\mathbb{N} \times \mathbb{N}$, the set of ordered pairs of natural numbers. Let R be a relation on $\mathbb{N} \times \mathbb{N}$ which is defined by $(a, b) R (c, d)$ if and only if $ad = bc$. Prove that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$. 5
- (d) Show that if a and b are any two elements of a group G , then $(ab)^2 = a^2b^2$ if and only if G is abelian. 5
4. (a) Define normal subgroup. Show that a subgroup H of a group G is normal if and only if $xHx^{-1} = H \forall x \in G$. 1+4=5
- (b) Write down all the permutations on four symbols 1, 2, 3, 4. Which of these permutations are even? 3+2=5



(c) Consider the set $S = \{a, b, c\}$. Prove that $(P(S), \subseteq)$ is a poset. Draw the Hasse diagram of the poset $(P(S), \subseteq)$.
4+2=6

(d) Show that a lattice is a partially ordered set. 4

5. (a) Is the relation "in brother of" an equivalence relation on a set of all human beings? Why? 3

(b) If G is a group, then prove that $(ab)^{-1} = b^{-1}a^{-1} \forall a, b \in G$. 3

(c) Let R_0 denote the set of all non-zero real numbers. Prove that the map $f: R_0 \rightarrow R_0$ defined by

$$f(x) = \frac{1}{x} \quad \forall x \in R_0 \text{ is bijective.}$$

4

(d) Examine whether the following compound proposition is tautology or not.

$$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p) \quad 2$$

(e) Find the negation of each of the following quantified predicates:

$$2+2=4$$

(i) $(\exists x D)(x+2=7)$

(ii) $(\forall x \in D)(x+3 < 10)$,
where $D = \{1, 2, 3, 4\}$.

(f) Show that in a non-directed graph, the total number of odd degree vertices is even. 4

6. (a) Establish the equivalence:

$$p \rightarrow q \vee r \equiv p \wedge (\sim p) \rightarrow r \equiv p \wedge (\sim r) \rightarrow q$$

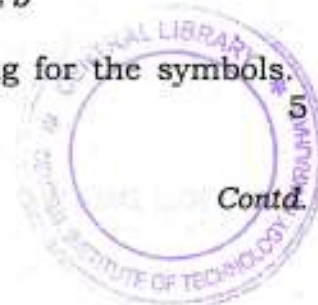
Hence write the following sentence in two different ways. If n is prime, then n is odd or n is 2. 3+2=5

(b) If L is a lattice, then for every $a, b \in L$ prove that—

(i) $a \vee b = b$ iff $a \leq b$;

(ii) $a \wedge b = a$ iff $a \leq b$

with usual meaning for the symbols.



- (c) Let A and B be two non-empty sets. If $f : A \rightarrow B$ is invertible, then show that f^{-1} is also invertible. 5
- (d) Prove that every cyclic group is abelian, but the converse may not be true in general. Give an example to show that every abelian group is not cyclic. 5

