53 (MA 302) DIMA

2019

DISCRETE MATHEMATICS

Paper : MA 302

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Let I be the set of all integers and define $a \equiv b\ (mod\ m)$ if $a - b$ is divisible by m, where $a, b \in I$ and m is a positive integer. Show that the congruence relation $\equiv$ is an equivalence relation on I. 6

   (b) If $f : A \to B$ and $g : B \to C$ are both bijective mappings, then prove that $g \circ f : A \to C$ is also bijective. Also show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. 5+3=8

Contd.

(c) Show that, the set $G\{L, \omega, \omega^2\}$ where ω is an cube root of unity, will form a group with respect to multiplication. 6

2. (a) Define normal subgroup. Show that, a subgroup H of a group G is normal if and only if $xHx^{-1} = H, \forall x \in G$. 2+5=7

(b) Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$, $n > 2$ by generating function method, given that $a_0 = 2$ and $a_1 = 3$. 7

(c) If R and S are two equivalence relation on a set A, then show that $R \cap S$ is also an equivalence relation on A. 6

3. (a) Define subgroup of a group. Show that intersection of two subgroups is again a subgroup of a group. 2+5=7

(b) Find the explicit formula for the sequence defined by the recurrence relation 6

$$a_n = 3a_{n-1} + 1, \quad a_1 = 5$$

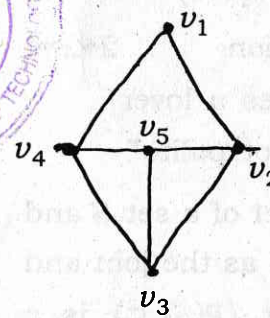
(c) Show that in a connected graph any two longest path have a common point. 7

4. (a) Prove that in any graph G, the total number of odd-degree vertices is even. 5

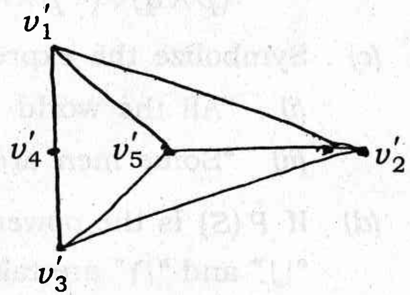
(b) Define vertex connectivity and edge connectivity of a graph. Prove that the edge connectivity of a graph G cannot exceed the minimum degree of a vertex in G. 2+5=7

(c) Prove that a non-empty connected graph G is Eulerian if and only if its vertices are all of even degree. 4+4=8

5. (a) Show that the graphs G_1 and G_2 are isomorphic 6



Graph → G_1



Graph → G_2

(b) Prove that, $(ab)^2 = a^2b^2$ if and only if G is an abelian group, where $a, b \in G$. 5

(c) If A, B, C are any three non-empty sets, then prove that

$$A \setminus (B \cup C) = (A \setminus B) \cap (A - C).$$ 4

(d) Obtain the conjunctive normal form of $\sim(p \vee q) \leftrightarrow (p \wedge q)$ 5

6\. (a) Let $Q(x, y)$ denote the statement $x = y + 3$. What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$? 4

(b) Obtain the principal disjunction normal form of the compound propositive

$$(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$$ 5

(c) Symbolize the expression 2+2=4

(i) "All the world loves a lover"

(ii) "Some men are not polite"

(d) If $P(S)$ is the power set of a set S and "$\cup$" and "$\cap$" are taken as the join and meet, then prove that $(P(S), \subseteq)$ is a lattice. 5

(e) Define Boolean algebra. Give an example of a Boolean algebra with justification. 2