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53 (MA 301) MATH–III

2021

MATHEMATICS–III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Evaluate : ***(any two)*** 4×2=8

(i) $L^{-1}\left\{\dfrac{3s+7}{s^2-2s-3}\right\}$

(ii) $L^{-1}\left\{\dfrac{e^{-5s}}{(s-2)^4}\right\}$

(iii) $L^{-1}\left\{\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$

(b) Solve : ***(any two)*** 3×2=6

(i) $\left(x^2-y^2-z^2\right)p+2xq=2xz$

(ii) $x^2p+y^2q=(x+y)z$

(iii) $x^2(z-y)p+y^2(x-z)q=z^2(y-x)$

(c) (i) If S_{ik} is symmetric and A_{ik} is skew-symmetric, then prove that $S_{ik}A_{ik}=0$. 3

(ii) If A_{ij} is skew-symmetric, then show that

$$\left(B^j_j\ B^m_n+B^i_n\ B^m_j\right)A_{im}=0.$$ 3

2. (a) When is a complex function analytic? Are the functions —

(i) $f(z)=e^{-y}\sin x-ie^{-y}\cos x$

(ii) $g(z)=e^{y}\cos x+ie^{y}\sin x$

analytic? Justify. 1+2+2=5

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(b) Form the partial differential equation of

(i) $z = f\left(x^2 + y^2\right)$

(ii) $f\left(x^2 - yz, y^2 - xz\right) = 0$ 2+3=5

(c) (i) Show that the process of contraction reduces a mixed tensor of rank 2 to a scalar. 2

(ii) Find the line element in spherical polar coordinate system. 4

(d) Evaluate the integral

$$\int_{(0,0)}^{(1,1)} \left(3x^2 + 4xy + 3y^2\right)dx + 2\left(x^2 + 3xy + 4y^2\right)dy$$

along the path —

(i) $y^2 = x$

(ii) $y = x^2$ 2+2=4

3. (a) Find Laplace transform of the following functions : ***(any four)*** 3×4=12

(i) $\left(t^2 + 3\right)^3 + 3t^2 + e^{-t}$

(ii) $e^{-t} \sin 3t$

(iii) $(t+2)^2 e^{2t}$

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(iv) $e^{-at}\cos \pi t$

(v) $t \sin at$

(b) Solve : ***(any one)*** 5

(i) $\dfrac{\partial^3 z}{\partial x^3} - 2\dfrac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$

(ii) $\dfrac{\partial^3 z}{\partial x^2} - 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = \sin x$

(c) If A^i and B_i are two arbitrary vectors then show that $A^i B_i$ is invariant. 3

4. (a) State the Cauchy's Integral formula. Evaluate the complex integral

$$\int_C \frac{4-3z}{z(z-1)(z-2)}$$

where C is the circle $|z| = \dfrac{3}{2}$. 1+5=6

(b) Solve : ***(any two)*** 3×2=6

(i) $z = p^2 + q^2$

(ii) $x^2 p^2 + y^2 q^2 = z^2$

(iii) $p^2 - q^2 = x - y$

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(c) (i) State and prove Quotient law of Tensors. 4

(ii) If $A_k^{ij} B_j C_j D^k$ is an invariant for arbitrary covariant vectors B_i, C_j and covariant vector D^k, show that A_k^{ij} is a mixed tensor of rank 3. 4

5. (a) Solve (by using Charpit's method) $\left(p^2+q^2\right)y = qz$. 6

(b) Define a harmonic function. Show that the function $u = x^2 - y^2$ is harmonic and find its conjugate harmonic function. 1+2+2=5

(c) Solve (using Laplace transform) $y'' + y = t$ if $y(0)=1$, $y'(0)=2$. 4

(d) (i) Assume $\phi = a_{jk} A^j A^k$. Then show that $\phi = b_{jk} A^j A^k$, where b_{jk} is symmetric. 2

(ii) If $a_{ijk}\, dx^i\, dx^j\, dx^k = 0$ for all values of a_{ijk}, then show that

$a_{123} + a_{132} + a_{231} + a_{213} + a_{312} + a_{321} = 0$. 3

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6. *(a)* If $C(m, n)$ is the co-factor of A_{mn} in $det\left(A_{mn}\right) = d \neq 0$ and $A^{mn} = \dfrac{C(m,n)}{d}$ then $A_{mn}\, A^{in} = \delta_m^i$. 2

(b) Expand $f(z) = \dfrac{4z+3}{z(z-3)(z+2)}$ in the region —

(i) $|z| < 1$

(ii) $2 < |z| < 3$ 1+2+2=5

(c) Determine the poles and residues of $f(z) = \dfrac{1}{z(z+1)^2(z+3)}$ and hence evaluate $\int_C f(z)\,dz$, where C is $|z| = 2$.

1+3+1=5

(d) If a and b are two constants and $L\{F(t)\} = f(s),\ L\{G(t) = g(s)\}$ then prove that $L\{aF(t) + bG(t)\} = af(s) + bg(s)$. 4

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(e) Solve 4

$$\frac{\partial^2 z}{\partial x^2 \partial y} = \sin x \sin y \text{ for which}$$

$$\frac{\partial z}{\partial y} = -2\sin y, \ x = 0 \text{ and } z = 0 \text{ where } y$$

is an odd multiple of $\frac{\pi}{2}$.

7. (a) Using the method of separation of variables, find the solution of

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0 \text{ when } u(x, 0) = 4e^{-x}.$$ 4

(b) Show that the function $f(z) = z^3$ is analytic everywhere in the complex plane. 4

(c) Find the fundamental metric tensor for the line element given by

$$ds^2 = dx^2 + dy^2 + dz^2 - 2dxdy + dydz - dzdx.$$

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(d) Show that—

$$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2}\log\frac{s^2 + b^2}{s^2 + a^2}.$$ 5

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(e) Find the Z-transform of : *(any one)* 2

(i) $a^{|k|}$

(ii) Unit impulse

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