Total number of printed pages–6

53 (MA 301) ENMA-III

2019

ENGINEERING MATHEMATICS – III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Form Partial differential equation : ***(any two)*** 3+3=6

   (i) $Z = f\left(x^2 + y^2\right)$

   (ii) $F = \left(x^2 - yz, y^2 - xz\right) = 0$

   (iii) $(x-a)^2 + (y-b)^2 + z^2 = a^2$

   (b) Define Laplace transform of a function $F(t)$ for $t > 0$. Evaluate : 2+2×3=8

   (i) $L\left\{\left(t^3+1\right)^3\right\}$

Contd.

(ii) $L\left\{3t^2 - e^{-2t} + 3\sin t\right\}$

(iii) $L\left\{\cos^2(3t)\right\}$

(c) If $a_{ijk}dx^i dx^j dx^k = 0$ for all values of a_{ij}, then show that

$$a_{123} + a_{132} + a_{213} + a_{231} + a_{312} + a_{321} = 0$$

6

2. (a) When a complex function is analytic? Is the function

$$f(z) = \frac{x}{x^2 + y^2} - i\frac{y}{x^2 + y^2}$$

analytic? Justify. 5

(b) Evaluate : 4×3=12

(i) $L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$

(ii) $L^{-1}\left\{\frac{e^{-3s}}{(s-2)^4}\right\}$

(iii) $L^{-1}\left\{\frac{6}{s-3} - \frac{3+4s}{s^2-16} + \frac{s-6}{s^2+9}\right\}$

(c) If A^i and B_i are two arbitrary vectors, then show that $A^i B_i$ is invariant. 3

3. (a) If $L\{F(t)\} = f(s)$ then prove that

$$L\left\{e^{at}F(t)\right\} = f(s-a),\ s > a.$$

5

(b) Solve : 5×2=10

(i) $q^2 = z^2 p\left(1 - q^2\right)$

(ii) $p^2 + q^2 = x^2 + y^2$

(c) Find the fundamental conjugate tensor for the line element given by

$$ds^2 = dx^2 + dy^2 + dz^2 - 2dxdy + dydz - dzdx$$

5

4. (a) Evaluate : $\int_0^{1+i}\left(x - y - ix^2\right)dz$

(i) along the path $y = x$

(ii) along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1+i$. 2+4=6

(b) Solve : ***(any one)*** 5

(i) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 3x^2 y$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x+y)$

(c) (i) Write down the two forms of Christoffel Brackets. 2

(ii) If A_{ij} is skew-symmetric, show that

$\left(B^i_j B^m_n + B^i_n B^m_j\right) A_{im} = 0$ 3

(d) Show that the function $f(z) = xy + iy$ is everywhere continuous but not analytic. 4

5. (a) Find the residues of

$$f(z) = \frac{z^3}{(z-1)^4 (z-2)(z-3)}$$

at its poles and hence evaluate

$\int_C f(z)dz$, where C is the circle $|z| = \frac{5}{2}$

6

(b) Evaluate $\int_C \log z dz$, where C is the unit circle $|z| = 1$. 4

(c) If $A^{ij}_k B_i C_j D^k$ is an invariant for arbitrary vectors B_i, C_j and D^k ; show that A^{ij}_k is a mixed tensor of rank 3. 5

(d) If $C(m,n)$ is the cofactor of A_{mn} in $det(A_{mn}) = d \neq 0$ and

$A^{mn} = \frac{C(m,n)}{d}$, then show that

$A_{mn} A^{in} = \delta^i_m$. 5

6. (a) Solve (Using Laplace Transform) :

$Y'' + Y = t, Y(0) = 1, Y'(0) = 2$ 5

(b) Solve : 5×2=10

(i) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos\theta$

(c) Using Cauchy's integral formula, evaluate

$$\int_C \frac{4-3z}{z(z-1)(z-2)} dz$$

where C is the circle $|z| = 3/2$. 5