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53 (MA 301) ENMA-III

2018

ENGINEERING MATHEMATICS–III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Prove that $L\{t\} = \frac{1}{s^2}, s > 0$. 5

(b) Find the fundamental conjugate tensor for the line element given by

$$ds^2 = dx^2 + dy^2 + dz^2 - 2dxdy + dydz - dzdx$$ 5

Contd.

(c) Show that the function $z|z|$ is not analytic anywhere. 5

(d) Using Charpit's method, solve : $2z + p^2 + qy + 2y^2 = 0$. 5

2. (a) Find Laplace transform of the following : ***(any two)*** 2×2=4

(i) $3t^2 - e^{-2t} + 3\sin t$

(ii) $\cos^2(3t)$

(iii) $(t^2 - 3)^2$

(b) If A_{ij} is antisymmetric and S_{ij} is symmetric, show that $A_{ij}S_{ij} = 0$. 4

(c) Solve : 4×3=12

(i) $x^2(y-z)\,p + y^2(z-x)\,q = z^2(x-y)$

(ii) $p(1-q^2) = q(1-z)$

(iii) $\sqrt{p} + \sqrt{q} = 2x - 3y$

3. *(a)* Determine the poles and residues of $f(z) = \frac{1-2z}{z(z-1)(z-2)}$ and hence evaluate $\int_C f(z)\,dz$ where C is $|z| = \frac{3}{2}$.

1+3+1=5

(b) If $a_{ijk}\,dx^i\,dx^j\,dx^k = 0$ for all values of a_{ij}, then show that

$$a_{123} + a_{132} + a_{213} + a_{231} + a_{312} + a_{321} = 0$$

6

(c) Evaluate : ***(any three)*** 3×3=9

(i) $L^{-1}\left\{\frac{6}{s-3} - \frac{3+4s}{s^2-16} + \frac{s-6s}{s^2+9}\right\}$

(ii) $L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$

(iii) $L\left\{t^2 \sin t\right\}$

(iv) $L\left\{e^{-t}\cos^2 t\right\}$

4. *(a)* Find the line element in cylindrical coordinates. 5

(b) Using Lagrange's method, solve : ***(any two)*** 3+3=6

(i) $(x-a)^2+(x-b)^2=z^2\cot^2\alpha$

(ii) $z=x^2 f(y)+y^2 g(x)$

(iii) $F\left(x^2+2yz,\ y^2+2zx\right)=0$

(c) Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in Laurent's series if

(i) $1<|z|<3$

(ii) $|z|<3$

(iii) $-3<|z|<3$

(iv) $|z|<1$ 5

(d) Evaluate the integral

$$\int_{(0,0)}^{(3,9)} (x+y)\,dx + (x^2 y)\,dy$$ along the path

(i) $y = x^2$ and (ii) $y = 3x$.

2+2=4

5. (a) Solve : ***(any one)*** 5

(i) $$4\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 16 \log (x+2y)$$

(ii) $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\, \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$$

(b) If A^i is an arbitrary contravariant vector and $C_{ij}A^iA^j$ is invariant, then show that C_{ij} is a covariant tensor of rank 2.

4

(c) Show that the function $u = 4xy - 3x + 2$ is harmonic. Construct the corresponding analytic function $f(z) = u(x, y) + iv(x, y)$. Express $f(z)$ in terms of complex variable z.

2+2+2=6

(d) Solve (Using Laplace transform) :

5

$$Y'' + Y = t,\ Y(0) = 1,\ Y'(0) = 2.$$

6. (a) Evaluate the complex integral $\int_C \frac{1}{(z^3 - 1)^2} dz$, where C is the circle $|z - 1| = 1$. 5

(b) By direct integration method, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

4

(c) If A_{ij} is skew-symmetric, show that $\left(B_j^i B_n^m + B_n^i B_j^m\right) A_{im} = 0$. 3

(d) Find *Z*-transform of : 2+2=4

(i) $\left\{a^{|k|}\right\}$

(ii) $\delta(k)=\begin{cases}1, & k=0\\ 0, & k\neq 0\end{cases}$

(e) Evaluate : 4

$$L^{-1}\left\{\frac{e^{-3s}}{(s-2)^4}\right\}$$