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53 (MA 301) ENMA III

2017

ENGINEERING MATHEMATICS III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Define Laplace transform of a function $F(t)$ for $t > 0$. Find Laplace transforms of the following functions :

   (i) $\cos(wt + \theta)$

   (ii) e^{3a-2bt}

   (iii) $\left(t^3 + 1\right)^3$

   where w, θ, a, b are constants. 2+2×3=8

(b) Solve : ***(any two)***

(i) $\left(x^2 - y^2 - z^2\right)p + 2xq = 2xz$

(ii) $x^2 p + y^2 q = (x + y)z$

(iii) $x^2(z - y)p + y^2(x - z)q = z^2(y - x)$

3+3=6

(c) (i) If S_{ik} is symmetric and A_{ik} is skew-symmetric, then prove that $S_{ik}A_{ik} = 0$ 3

(ii) If A_{ij} is skew-symmetric then show that $\left(B_j^j B_n^m + B_n^i B_j^m\right)A_{im} = 0$ 3

2. (a) When a complex function is analytic ? Is the function

$$f(z) = \frac{x}{x^2 + y^2} - i\frac{y}{x^2 + y^2}$$

analytic ? Justify. 5

(b) Form the partial differential equation from :

(i) $Z = f\left(x^2 + y^2\right)$

(ii) $F\left(x^2 - yz,\ y^2 - xz\right) = 0$ 2+3=5

(c) (i) Show that the process of contraction reduces a mixed tensor of rank 2 to a scalar. 2

(ii) Find the line element in spherical polar coordinate system. 4

(d) Evaluate the integral

$$\int_{(0,0)}^{(1,1)} \left(3x^2 + 4xy + 3y^2\right)dx + 2\left(x^2 + 3xy + 4y^2\right)dy$$

along the path (i) $y^2 = x$ (ii) $y = x^2$

2+2=4

3. (a) Evaluate : 3×4=12

(i) $L\left\{-3{\cdot}8t^2 e^{-2{\cdot}4t}\right\}$

(ii) $L\left\{e^{-kt}\left(a\cos t + b\sin t\right)\right\}$

(iii) $L^{-1}\left\{\dfrac{4s+12}{s^2+8s+16}\right\}$

(iv) $L^{-1}\left\{\dfrac{e^{-3s}}{(s-2)^4}\right\}$

(b) Solve : ***(any one)*** 5

(i) $$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$$

(ii) $$\frac{\partial^3 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$$

(c) If A^i and B_i are two arbitrary vectors, then show that $A^i B_i$ is invariant. 3

4. (a) State the Cauchy's Integral formula. Evaluate the complex integral

$$\int_C \frac{e^{i\pi z}}{2z^2 - 5z + 2} dz$$

where C is the circle $|z| = 1$.

1+5=6

(b) Solve : ***(any two)*** 3+3=6

(i) $z = p^2 + q^2$

(ii) $x^2 p^2 + y^2 q^2 = z^2$

(iii) $p^2 - q^2 = x - y$

(c) (i) State and prove Quotient law of Tensors. 4

(ii) If $A_k^{ij} B_i C_j D^k$ is an invariant for arbitrary covariant vectors B_i, C_j and covariant vector D^k, show that A_k^{ij} is a mixed tensor of rank 3. 4

5. (a) Solve (by using Charpit's method)

$$\left(p^2 + q^2\right) y = qz$$ 6

(b) Define a harmonic function. Show that the function $u = x^2 - y^2$ is harmonic and find its conjugate harmonic function. 1+2+2=5

(c) Find Z-transform of :

(i) Unit impulse

$$\delta(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

(ii) Discrete unit step

$$U(k) = \begin{cases} 0, & k < 0 \\ 1, & k \geq 0 \end{cases}$$

(iii) $\left\{a^{|k|}\right\}$ 1+1+2=4

(d) (i) Assume $\varphi = a_{jk}A^j A^k$. Then show that $\varphi = b_{jk}A^j A^k$, where b_{jk} is symmetric. 2

(ii) If $a_{ijk}dx^i dx^j dx^k = 0$ for all values of a_{ijk}, then show that

$$a_{123} + a_{132} + a_{231} + a_{213} + a_{312} + a_{321} = 0$$

3

6. (a) If $C(m, n)$ is the cofactor of A_{mn} in $det.(A_{mn}) = d \neq 0$ and $A^{mn} = \frac{C(m, n)}{d}$ then $A_{mn}A^{in} = \delta^i_m$. 2

(b) Expand $f(z) = \frac{4z+3}{z(z-3)(z+2)}$ in the region

(i) $|z| < 1$ (ii) $2 < |z| < 3$ 1+2+2=5

(c) Determine the poles and residues of

$$f(z) = \frac{1}{z(z+1)^2(z+3)}$$ and hence

evaluate $\int_C f(z)dz$ where C is $|z| = 2$.

1+3+1=5

(d) Solve : (Using Laplace transform)

$Y'' + Y = t, \ Y(0) = 1, \ Y'(0) = 2$ 4

(e) Solve :

$$\frac{\partial^2 z}{\partial x^2 \partial y} = \sin x \sin y$$ for which

$$\frac{\partial z}{\partial y} = -2\sin y, \ x = 0 \text{ and } z = 0$$

where y is an odd multiple of $\frac{\pi}{2}$.

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