53 (MA 301) ENMA-III

2017

ENGINEERING MATHEMATICS-III

Paper : MA-301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Show that $f(z)=\log_e z$ is analytic everywhere in the complex plane except at origin. 5

   (b) Form the partial differential equation from : ***(any two)*** 3+3=6

   (i) $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$

   (ii) $z=x^2 f(y)+y^2 g(x)$

   (iii) $F\left(xy+z^2, x+y+z\right)=0$

(c) Define Laplace transform of a function $F(t)$. If c_1 and c_2 are two constants and $L\{F_1(t)\}=f_1(s)$, $L\{F_2(t)\}=f_2(s)$ then show that

$L\{c_1 F_1(t)+c_2F_2(t)\}=c_1f_1(s)+c_2f_2(s)$. 1+3=4

(d) Show that skew-symmetric tensor of rank 2 has almost $\frac{n(n-1)}{2}$ different components in n-dimensional space. 5

2. (a) Using Laplace transform solve

$y''+y=t$; $y(0)=1$, $y'(0)=2$

with usual meaning of the symbols. 5

(b) By the method of integration solve

$$\frac{\partial^2 z}{\partial x^2}+z=0$$

given that $z=e^y$ and $\frac{\partial y}{\partial x}=1$ when $x=0$. 5

(c) A covariant tensor has components $2x, y^2, z^2x$ in rectangular coordinates. Determine its covariant components in cylindrical coordinates. 5

(d) Expand $f(z)=\frac{1}{(z-1)(z-2)}$ in the region

(i) $|z|>2$

(ii) $0<|z-1|<1$

2+3=5

3. (a) Evaluate $\int_0^{2+i}(\bar{z})^2\,dz$ along the path $y=0$ and $x=2$. 2+2=4

(b) Solve the following equations : ***(any two)***

(i) $\frac{y^2z}{x}p+xzq=y^2$

(ii) $\left(x^2-y^2-z^2\right)p+2xyq=2xz$

(iii) $x\left(y^2-z^2\right)p+y\left(z^2-x^2\right)q-z\left(x^2-y^2\right)=0$

3+3=6

(c) If $L\{F(t)\}=\frac{e^{-\frac{1}{s}}}{s}$ find $L\left\{e^{-t}F(3t)\right\}$. 5

(d) Find the line element in cylindrical coordinates. 5

4. (a) Prove that the function $f(z)=\sqrt{|xy|}$ is not analytic at origin even though Cauchy-Riemann equations are satisfied at origin. 6

(b) Show that δ_{ij} is a covariant tensor of rank 2 and hence show that $\delta_{ij}dx^i dx^j$ is invariant. 4

(c) Find inverse Laplace transform of $\frac{e^{-5s}}{(s-2)^4}$. 4

(d) Using Charpit's method solve $2xz-px^2-2qxy+pq=0$ 6

5. (a) Evaluate : 3+3=6

(i) $L\{t\sinh(at)\}$

(ii) $L\left\{e^{-t}\cos^2 t\right\}$

(b) Show that the inner product of A_q^p and B_k^{ij} is a tensor of rank three. 4

(c) Using the method of separation of variables, solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ where $u(x,0)=4e^{-x}$. 5

(d) Evaluate $\int_C \frac{e^z}{z^2+1}dz$ 5

over the circular path $|z|=2$.

6. (a) Using Laplace transform show that

$$\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}dt=\log 2$$ 4

(b) Solve the following equations : 4+4=8

(i) $q^2=z^2p^2\left(1-p^2\right)$

(ii) $z=p^2+q^2$

(c) What do you mean by contraction ? Show that contraction reduces a mixed tensor of rank 2 to a scalar. 4

(d) Using Cauchy's integral formula evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}dz$

where C is the circle $|z|=\frac{3}{2}$. 4

7. (a) Find the residues of

$$f(z)=\frac{z^3}{(z-1)^4(z-2)(z-3)}$$ and hence

evaluate $\int_C f(z)dz$ where C is the circle

$|z|=\frac{5}{2}$. 6

(b) State and prove the Quotient law of tensors. 5

(c) Solve the following :

(i) $ty''+y'+4ty=0$ if $y(0)=3, y'(0)=0$ 5

(ii) $(D^2+9)y=\cos 2t$ if $y(0)=1$, $y\left(\frac{\pi}{2}\right)=-1$ 4