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53 (MA 301) ENMA III

2015

ENGINEERING MATHEMATICS III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer Q. No. ***1*** *and* ***any four*** *from the rest.*

1. (a) Fill in the blanks : 1×5=5

   (i) The poles of $\frac{z}{\sin z}$ are given by ________, $n \in z \cdot (2n\pi / n\pi / 2n\pi i)$

   (ii) $|z - z_0| = r$ represents a/an ________. (ellipse / circle / parabola)

(iii) The value of $\sqrt{-a} \times \sqrt{-b}$ is ________. $(-\sqrt{ab} / \sqrt{-ab}\, \sqrt{iab})$

(iv) The Cauchy's integral formula is used for ________ (indefinite integral / definite integral / definite and indefinite integral)

(v) If $f(z)$ and $g(z)$ are anlytic functions, then $f+g$ is ________ (analytic / differentiable / continuous / analytic, differentiable and continuous).

(b) Answer the following questions :

(i) Define Laplace transform of a function $f(t)$. 1

(ii) Define non-linear partial differential equation. 1

(iii) Write the general form of Lagrange's equation. 1

(iv) $L\{e^{a+}\} = \frac{1}{s-a}$, $s < a$ (True / False) 1

(v) Write the relation between the coordinates of the Cartesian system and spherical polar system. 1

(vi) What method is generally used to solve non-linear partial differential equation having three variables ? 1

(vii) Write the nth order linear homogeneous partial differential equation. 1

(viii) Define Kronecker Delta and hence prove that $\delta^i_j A^j = A^i$. 2

(ix) What is an invariant ? Give an example. 2

(x) Find $L\{sint.cost\}$ and $L\{(t+2)^2 e^t\}$. 2+2

2. (a) Show that the function $f(z) = xy + iy$ is everywhere continuous but not analytic. 5

(b) If $L\{F(t)\} = f(s)$, then show that $L\{e^{at} F(t)\} = f(s-a)$, $s > a$. 5

(c) (i) Write down the two forms of Christoffel Brackets. 2

(ii) If $a_{ij} dx^i dx^j = 0$, $\forall a_{ij}$, show that $a_{12} + a_{21} = 0$. 3

(d) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$. 5

3. (a) Find $L\{ F(t) \}$ if $F(t) = 0$ for $0 < t < 2$

$= 4$ for $t > 2$ 5

(b) Form partial differential equation : ***(any two)*** 3+3=6

(i) $z = ax + by + a^2 + b^2$

(ii) $z = yf(x) + xg(y)$

(iii) $F\left(x+y+z,\, x^2+y^2+z^2\right)=0$

(c) Determine the analytic function $f(z)=u+iv$ if $v=\log\left(x^2+y^2\right)+x-2y$. 4

(d) (i) If $\overline{A}^i=\frac{\partial \overline{x}^i}{\partial x^r}A^r$, show that

$$A^K=\frac{\partial x^K}{\partial \overline{x}^i}\overline{A}^i,$$ 2

(ii) If δ_{ij} is symmetric and A_{ij} is skew symmetric, then show that $\delta_{ij}A_{ij}=0$. 3

4. (a) Find the residues of

$$f(z)=\frac{z^3}{(z-1)^4(z-2)(z-3)}$$ at its poles

and hence evaluate $\int_C f(z)dz$, where C

is the circle $|z|=\frac{5}{2}$. 6

(b) Find the line element in cylindrical system. 4

(c) Using Charpit's method, solve $(p^2+q^2)y = qz$. 6

(d) Find $L\{(1+te^{-t})^3\}$ 4

5. (a) Find Z-transform of 2×3=6

(i) $\{a^{|K|}\}$

(ii) $\left\{\frac{1}{2K}\right\}, -4 \le K \le 4$

(iii) Unit impulse

$$\delta(K) = \begin{cases} 1, & K = D \\ 0, & K \neq D \end{cases}$$

(b) Solve : ***(any one)*** 4

(i) $\frac{y^2 z}{x} p + xzq = y^2$

(ii) $x(y-z)p + y(z-x)q = z(x-y)$

(c) Evaluate the integral $\int_c \frac{z}{z^2+1} dz$, where

c is the curve $\left|z+\frac{1}{z}\right| = 2$. 6

(d) If $C(m,n)$ is the cofactor of A_{mn} in

$det(A_{mn}) = d \neq 0$, and $A^{mn} = \frac{C(m,n)}{d}$,

show that $A_{mn} A^{in} = \delta^i_m$. 4

6. (a) Solve : ***(any one)***

(i) $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos\theta$ 4

(b) Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$ 4

(c) Evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$ 5

Or

Solve the differential equation (Using Laplace Transform) : 5

$$Y''+Y=t\ ,\ \ Y(0)=1,\ \ Y'(0)=2$$

(d) Show that the metric tensor is a symmetric covariant tensor of rank 2. 4

(e) If A_{ij} is skew symmetric, show that

$$\left(B_j^i B_n^m + B_n^i B_j^m\right)A_{im}=0$$ 3
