53 (MA 301) ENMA III

2014

ENGINEERING MATHEMATICS-III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any 5 (five)** questions.*

1. *(a)* Form partial differential equations from the following : 2+3=5

   (i) $(x-a)^2+(y-b)^2+z^2=c^2$

   (ii) $f\left(x^2+2yz,\, y^2+2zx\right)=0$

   (b) If $a_{ijk}dx^i dx^j dx^k=0$ for all values of a_{ij}, then show that

   $a_{123}+a_{132}+a_{213}+a_{231}+a_{312}+a_{321}=0$ 6

(c) Show that the function $\overline{u}(x, y) = e^{-2xy} \sin\left(x^2 - y^2\right)$ is harmonic, find the conjugate function $v(x, y)$ and express $u + iv$ as an analytic function of z. 2+3=5

(d) Evaluate the integral $\int_c \log z dz,$ where c is the unit circle $|z| = 1$. 4

2. (a) Find Laplace transform of the following functions :

(i) e^{3a-2bt} (ii) $\cos(wt + \theta)$

(iii) $\left(t^2 - 3\right)^2$ (iv) $(t+1)^3$,

where a, b, w, θ are constants. 2×4=8

(b) Solve :

(i) $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$

(ii) $z^2 = 1 + p^2 + q^2$ 4+4=8

(c) If A_{ij} is antisymmetric and S_{ij} is symmetric, show that $A_{ij}S_{ij=0}$ 4

3. (a) Solve :

(i) $$\frac{\delta^3 z}{\delta x^3}-4\frac{\delta^3 z}{\delta x^2\delta y}+5\frac{\delta^3 z}{\delta x\delta y^2}-2\frac{\delta^3 z}{\delta y^3}=e^{2x+y}$$

(ii) $$\frac{\delta^2 z}{\delta y^2}+3\frac{\delta^2 z}{\delta x\delta y}+2\frac{\delta^2 z}{\delta y^2}=24xy$$ 4+3=7

(b) Find Laplace transform of the following functions : 2×3=6

(i) $e^{-3t}\cos\pi t$

(ii) $e^{-Kt}(a\cos t+b\sin t)$

(iii) $e^{-t}\left(a_0+a_1t+a_2t^2+\ldots+a_nt^n\right)$

(c) (i) Evaluate $\oint_c \frac{\cos z}{z-\pi}dz$, where c is the circle.

$|z-1|=3$ 3

(ii) find the sum of residues of the function $f(z)=\frac{\sin z}{z \cos z}$ at its pole inside the circle $|z|=2$. 4

4. (a) If $A_K^{ij} B_i C_j D^k$ is an invariant for arbitrary vectors B_i, C_j and D^k; Show that A_k^{ij} is a mixed tensor of rank 3. 5

(b) Determine the solution of one-dimensional heat equation $\frac{\delta^2 u}{\delta x^2}=\frac{1}{c^2}\frac{\delta u}{\delta t}$ subject to the boundary conditions $u(0,t)=u(l,t)=0$ $(t>0)$ and the initial condition $u(x,0)=x$, l being the length of the *bar*. 10

(c) If $C(m,n)$ is the cofactor of A_{mn} in $\det(A_{mn})=d \neq 0$ and $A^{mn}=\frac{C(m,n)}{d}$, then show that $A_{mn}A^{in}=\delta_m^i$. 5

5. *(a)* Evaluate $\int_0^{3+i} z^2 dz$ along

(i) the line $y = x/3$

(ii) along $x = 3y^2$

Does the integration depend upon the path ?

2+2+1=5

(b) Evaluate : 3×4=12

(i) $L^{-1}\left\{\frac{6}{2S-3} - \frac{3+4S}{9S^2-16} + \frac{8-6S}{16S^2+9}\right\}$

(ii) $L^{-1}\left\{\frac{4S+12}{S^2+8S+16}\right\}$

(iii) $L^{-1}\left\{\frac{e^{-3S}}{(S-2)^4}\right\}$

(iv) $L^{-1}\left\{\log\left(1+\frac{1}{S^2}\right)\right\}$

(c) Assume $\phi = a_{jk} A^j A^k$. Then show that $\phi = b_{jk} A^j A^k$, where b_{jk} is symmetric. 3

6. (a) Solve $\dfrac{\delta^2 z}{\delta x \delta y} = Sin\, x\, Sin\, y$ for which $\dfrac{\delta z}{\delta y} = -2 Sin\, y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\pi/2$. 5

(b) State and prove Quotient law of tensors. 6

(c) (i) Show that $f(z) = log\, z$ is analytic everywhere in the complex plane except at origin. 5

(ii) Expand in Laurent's series

$$f(z) = \frac{1}{z(z-1)(z-2)} \text{ for } |z-1| < 1.$$

4

7. *(a)* Find the Line element in the spherical polar coordinates. 5

(b) Evaluate the complex integration by using Cauchy's integral formula

$$\int_c \frac{3z^2+z+1}{\left(z^2-1\right)(z+3)}\,dz,$$

where c is the circle $|z|=2$. 5

(c) Solve (using Laplace transform) :

$$Y'' + 9Y = \cos 2t,\ Y(0)=1,\ Y(\pi/2) = -1$$

5

(d) Find Z- transform of

(i) Unit impulse

$$\delta(k) = \begin{cases} 1, & k=0 \\ 0, & k \neq 0 \end{cases}$$

(ii) $\left\{a^{|k|}\right\}$

(iii) Discrete unit step

$$U(k)=\begin{cases}0, & k<0\\ 1, & k\geq 0\end{cases}$$

1+3+1=5
