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53 (MA 301) ENMA

2019

ENGINEERING MATHEMATICS-III

Paper : MA 301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Find Laplace transform of the following functions : ***(any two)*** 3×2=6

   (i) $\sin^2 8t$

   (ii) $3t^4 e^{-0.5t}$

   (iii) $e^{-kt}(\cos t + \sin t)$

   (iv) $t \sin(at)$

Contd.

(b) Form partial differential equations of the following : ***(any two)*** 3×2=6

(i) $(x-a)^2+(y-b)^2+z^2=c^2$

(ii) $z=x^2 f(y)+y^2 g(x)$

(iii) $f\left(x^2-2yz,\, y^2-2xz\right)=0$

(c) (i) If S_{ik} is symmetric and A_{ik} is anti-symmetric then show that :

$S_{ik}A_{ik}=0$ 3

(ii) If A_{ij} is skew-symmetric, show that

$\left(B_j^i B_n^m + B_n^i B_j^m\right)A_{im}=0$ 2

(d) Find the image of $|z-3i|=3$ under the mapping $w=\dfrac{1}{Z}$. 3

2. (a) When is a complex function analytic ? Are the functions

(i) $f(z)=e^{-y}\sin x - ie^{-y}\cos x$

(ii) $g(z)=e^{y}\cos x + ie^{y}\sin x$

analytic ? Justify. 1+2+2=5

(b) Solve : ***(any one)*** 5

(i) $\dfrac{\partial^2 z}{\partial x^2}-2\dfrac{\partial^2 z}{\partial x \partial y}+\dfrac{\partial^2 z}{\partial y^2}=x^2+y$

(ii) $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial x \partial y}-6\dfrac{\partial^2 z}{\partial y^2}=y\cos x$

(c) (i) If $C(m,n)$ is the cofactor of A_{mn} in det $(A_{mn})=d\neq 0$ and $A^{mn}=\dfrac{C(m,n)}{d}$, then show that $A_{mn}A^{in}=\delta_m^i$ 4

(ii) Show that the process of contraction reduces a mixed tensor of rank 2 to a scalar. 2

(d) Evaluate the integral :

$$\int_{(0,0)}^{(1,1)} \left(3x^2+4xy+3y^2\right)dx+2\left(x^2+3xy+4y^2\right)dy$$

along the path

(i) $y^2=x$

(ii) $y=x^2$ 2+2=4

3. (a) Evaluate : ***(any two)*** 3×2=6

(i) $L^{-1}\left\{\frac{3s+2}{s^3}-\frac{2s-18}{s^2+9}+\frac{4-3s}{16s^2+9}\right\}$

(ii) $L^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\}$

(iii) $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$

(b) Using Charpit's method solve the partial differential equation : 6

$$\left(p^2+q^2\right)y = qz$$

(c) If $\overline{A}^{ij} = A^{\alpha\beta}\frac{\partial \overline{x}^i}{\partial x^\alpha}\frac{\partial \overline{x}^j}{\partial x^\beta}$ then show that

$$A^{pq} = \overline{A}^{ij}\frac{\partial x^p}{\partial \overline{x}^i}\frac{\partial x^q}{\partial \overline{x}^j}$$ 4

(d) If $L\{F(t)\} = f(s)$, prove that

$$L\{F(at)\} = \frac{1}{a}f\left(\frac{s}{a}\right)$$ 4

4. (a) Solve the following partial differential equations : ***(any two)*** 4×2=8

(i) $\frac{y^2z}{x}p + xzq = y^2$

(ii) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

(iii) $xp - yq = y^2 - x^2$

(b) State the Cauchy's Integral formula. Evaluate the complex integral

$$\int_C \frac{4-3z}{z(z-1)(z-2)}$$

where C is the circle $|z| = \frac{3}{2}$. 1+4=5

(c) Assume $\phi = a_{jk}A^jA^k$. Then show that $\phi = b_{jk}A^jA^k$, where b_{jk} is symmetric. 3

(d) If $L\{F(t)\} = \frac{e^{-\frac{1}{s}}}{s}$, find $L\{e^{-t}F(3t)\}$. 4

5. (a) Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, where $x = 0$,

$\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$ 5

(b) Define a harmonic function. Show that the function $u = x^2 - y^2$ is harmonic and find its conjugate harmonic function. 1+2+2=5

(c) Show that

$$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2} \log \frac{s^2 + b^2}{s^2 + a^2}$$ 5

(d) Find the fundamental metric tensor for the line element given by

$$ds^2 = dx^2 + dy^2 + dz^2 - 2dxdy + dydz - dzdx$$ 5

6. (a) Expand : $f(z) = \frac{4z+3}{z(z-3)(z+2)}$ in the region (i) $|z|<|$ (ii) $2<|z|<3$ 5

(b) If $A_k^{ij} B_i C_j D^k$ is invariant for arbitrary vectors B_i, C_j and D^k, then show that A_k^{ij} is a mixed tensor of rank 3. 4

(c) Solve : ***(any one)*** 4

(i) $p(1+q) = qz$

(ii) $p^2 + q^2 = x^2 + y^2$

(d) Find z-transform of the following functions : 3+2=5

(i) $f(k) = \begin{cases} 5^k, & k < 0 \\ 3^k, & k \geq 0 \end{cases}$

(ii) $f(k) = \left(\frac{1}{2}\right)^{|k|}$

(e) Define singularity and pole of a complex function. 2

7. (a) If $a_{ijk}\, dx^i\, dx^j\, dx^k = 0$ for all values of a_{ijk} then show that 4

$$a_{123} + a_{132} + a_{213} + a_{231} + a_{312} + a_{321} = 0$$

(b) Using the method of separation of variables, find the solution of 4

$$3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0 \text{ when } u(x,0)=4e^{-x}$$

(c) Determine the poles and residues of

$$f(z)=\frac{1}{z(z+1)^2(z+3)}$$

and hence evaluate $\int_C f(z)dz$ where C is $|z|=2$. 1+3+1=5

(d) Show that the function $f(z)=z^3$ is analytic everywhere in the complex plane. 4

(e) If A^i and B_i are two arbitrary vectors then show that A^iB_i is invariant. 3