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53 (MA 301) ENMA III

2013

(December)

ENGINEERING MATHEMATICS-III

Paper : MA-301

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer any five questions.

1. *(a)* Form the partial differential equation from : 3+3=6

   (i) $z = ax + by + a^2 + b^2$

   (ii) $F\left(x^2 + 2yz,\ y^2 + 2zx\right) = 0$

   (b) If $z = \begin{cases} \dfrac{x^2 y(y - ix)}{x^6 + y^2}; & z \neq 0 \\ 0 & ; \ z = 0 \end{cases}$ 5

   then show that $f(z)$ is not analytic at $z = 0$

Contd.

(c) Define Laplace transform of the function $F(t)$. If $L\{F(t)\}=f(s)$, then prove that

$$L\{F(at)\}=\frac{1}{a}f(s/a).$$ 1+5=6

(d) If Aij is a skew symmetric tensor of rank two, prove that $\left(\delta_j^i\delta_l^k+\delta_l^i\delta_j^k\right)_{Aik}=0$

3

2. (a) Determine a, b, c and d so that the function

$$f(z)=(x^2+axy+by^2)+i(cx^2+dxy+y^2)$$

is analytic. 5

(b) Using charpit's method solve 5

$$2z+p^2+qy+2y^2=0$$

(c) Evaluate 3+2=5

(i) $L^{-1}\left\{\frac{e^{-3S}(2S+7)}{S^2+16}\right\}$

(ii) $L^{-1}\left\{\frac{12}{(S-3)(S+1)}\right\}$

(d) Prove that the metric tensor is a symmetric covariant tensor of rank 2. 5

3. (a) If $L\{F(t)\} = f(s)$, then prove that 5

$L\{F^1(t)\} = sf(s) - F(0)$, if

(i) F is continuous for $0 \le t \le N$

(ii) $F(t)$ is of exponential order for $t > N$

(iii) $F^1(t)$ is sectionally continuous for $0 \le t \le N$.

(b) Evaluate the complex integral

$\int_C \frac{z}{(z^2 - 3z + 2)} dz$, where C is the circle

$|z - 2| = \frac{1}{2}$ 6

(c) Solve : 4

$x(y-z)p + y(z-x)q = z(x-y)$

(d) Assume $A^{ijk} B_{ij}^{p} = C^{pk}$, where B_{ij}^{p} is an arbitrary tensor and C^{pk} is a contravariant tensor of rank two. Show that A^{ijk} is a contravariant tensor of rank three. 5

4. (a) If A^i is an arbitrary contravariant vector and $C_{ij}A^iB^j$ is an invariant, then show that $(C_{ij}+C_{ji})$ is a covariant tensor of rank two. 5

(b) Given that $u(x,y)=x^2-y^2$ and

$$v(x,y)=-\frac{y}{x^2+y^2}$$

Prove that both u and v are Harmonic function but $u+iv$ is not analytic function z.

3+3=6

(c) Find laplace transform of the following functions : 2+2=4

(i) $3t^2-e^{-2t}+3\,sin\,t$

(ii) $cos^2(3t)$

(d) Solve : $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$ 5

5. (a) Solve : 4

$$z^2=1+p^2+q^2$$

(b) Evaluate the integral

$$\int_0^{1+i}\left(x-y+ix^2\right)dz$$ 6

(a) Along the straight line from

$z = 0$ to $z = 1 + i$

(b) Along the imaginary axis from $z = 0$ to $z = i$ and then along a line parallel to real axis from $z = i$ to $z = 1 + i$

(c) (i) Find z-transform of the discrete unit step function 3

$$U(K)=\begin{cases}0 & ; \quad K<0 \\ 1 & ; \quad K\geq 0\end{cases}$$

(ii) Find z-transform of the sequence 2

$$\left\{\frac{1}{2K}\right\}; \quad -4\leq K\leq 4$$

(d) If $C(m, n)$ is the co-factor of A_{mn} in

$det\left(A_{mn}\right)=d\neq 0$ and $A^{mn}=\dfrac{C(m,n)}{d}$,

then show that $A_{mn}A^{in}=\delta_m^i$, further show that $A_{mn}A^{mn}=\eta$ 4+1=5

6. (a) Find the solution of the wave equation

$$\frac{\partial^2 y}{\partial x} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$ with the boundary conditions $y(0,t) = y(l,t) = 0$ and the initial conditions $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{e}\right)$ and $\left.\frac{\partial y}{\partial t}\right]_{t=0} = 0$ 10

(b) If the metric is given by 5

$$ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$$

find the Conjugate metric tensor g^{ij}

(c) Determine the region in the *w*-plane corresponding to region bounded by the lines $x = 0, y = 0, x = 2, y = 4$ in *z*-plane under the transformation $W = 2\left(e^{\frac{i\pi}{3}}\right)z$. 5

7. (a) Solve $\frac{\partial^2 z}{\partial x^2} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ When $x = 0$ and $z = 0$ when y is odd multiple of $\pi/2$. 4

(b) Find the Taylor's series which represent the function $\frac{z^2-1}{(z+2)(z+3)}$ when (i) $2<|z|<3$

(ii) $|z|>3$

6

(c) Find the line element in spherical-polar coordinates. 5

(d) Find the Laplace transform of 2+3=5

(i) $(t+2)^3$

(ii) $(sint-cost)^2$

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