Total number of printed pages–5

53 (MA 201) MATH

2021

ENGINEERING MATHEMATICS-II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* If u and v are functions of x and y, then show that

$$\frac{\partial (u, v)}{\partial (x, y)} \times \frac{\partial (x, y)}{\partial (u, v)} = 1 .$$ 6

(b) Find the value of $\left\lceil \frac{1}{2} \right.$. 7

(c) Let X be a random variable with probability density function $f(x) = c(1-x)$; $0 < x < 1$. Find c, $E(X)$ and $V(X)$. 2+2+3=7

Contd.

2. *(a)* If two dice are thrown together, then find the probability of getting either at least one 6 or a sum of 8. 5

(b) Find the Fourier series for the function $f(x) = x$ in the interval $-\pi < x < \pi$. 8

(c) Show that,

$$\beta(m, n) = 2\int_0^{\pi/2} sin^{2m-1}\theta . cos^{2n-1}\theta \, d\theta \quad 7$$

3. *(a)* Find median and mode from the following data :

Class Interval	0–10	10–20	20–30	30–40	40–50
Frequency	5	6	8	30	10

4+4=8

(b) Define Poisson distribution. Show that in case of Poisson distribution, mean and variance are equal. 2+6=8

(c) Find the rank of the matrix *A*, where

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{pmatrix} \quad 4$$

4. *(a)* Show that, $\vec{\nabla}^2\left(\frac{x}{r^3}\right)=0$, where

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $|\vec{r}|=r$. 6

(b) Find the work done when a force $\vec{F}$, where $\vec{F}=\left(x^2-y^2+x\right)\hat{i}-(2xy+y)\hat{j}$, moves a particle in xy–plane from (0, 0) to (1, 1) along the parabola $y^2=x$. 6

(c) Show that the diagonal elements of a Hermitian matrix are real. 3

(d) Show that $\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$ is not a nilpotent matrix of order 3. 5

5. *(a)* Evaluate,

$$\iint_S \{(x+z)dydz+(y+z)dzdx+(x+y)dxdy\},$$

where S is the surface of the sphere $x^2+y^2+z^2=4$. 4

(b) Compute the inverse of the following matrix by using elementary row transformation :

$$\begin{pmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{pmatrix}$$ 6

(c) If A is a real skew-symmetric matrix such that $A^2 + I = 0$, show that A is orthogonal and is of even order. 2+3=5

(d) Find the unit normal vector to the level surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$. 5

6. (a) Reduce the matrix A to its normal form, where

$$A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

and hence find its rank. 6+1=7

(b) If $\vec{a}$ is a constant vector, show that

$curl\,(\vec{r} \times \vec{a}) = -2\vec{a}$,

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 2

(c) Determine the constant 'a' so that the vector

$\vec{v} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$

is solenoidal. 2

(d) Show that a real 2×2 normal matrix is either symmetric or the sum of a scalar matrix and a skew-symmetric matrix. 5

(e) Find the value of x, y, z, s and t, if

$$A = \begin{pmatrix} x & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & y \\ z & s & t \end{pmatrix}$$ is orthogonal. 4