Total number of printed pages–7

53 (MA 201) MATH-II

2018

ENGINEERING MATHEMATICS–II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Find the angle of intersection at (4, –3, 2) of the surface $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$. 5

(b) Show that $\vec{\nabla}^2\left(\frac{x}{r^3}\right) = 0$,

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$. 6

Contd.

(c) Find the work done when a force $\vec{F}$, where $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$, moves a particle in xy-plane from (0, 0) to (1, 1) along the parabola $y^2 = x$. 5

(d) Evaluate :

$$\iint_S \{(x+z)\,dy\,dz + (y+z)\,dz\,dx + (x+y)\,dx\,dy\}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 4$. 4

2\. (a) If A is real skew-symmetric matrix such that $A^2 + I = 0$, show that A is orthogonal and is of even order. 2+3=5

(b) Show that $\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$ is not a nilpotent matrix of order 3. 5

(c) Compute the inverse of the following matrix using elementary row transformation :

$$\begin{pmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{pmatrix}$$ 6

(d) Find the rank of the matrix A where

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{pmatrix}$$ 4

3. (a) Calculate the mean and variance of Poisson distribution if its probability mass function is $P(X = x) = \dfrac{e^{-a} \cdot a^{x}}{\lfloor x}$;

where $x = 0, 1, 2, \ldots \infty$ and 'a' is the parameter of the distribution. 6

(b) Evaluate the distribution function for the following probability density function

$$f(x)=\begin{cases}\dfrac{x}{3} & ; \quad 0<x\leq 1\\ \dfrac{5}{27}(4-x) & ; \quad 1<x\leq 4\\ 0 & ; \quad \text{elsewhere}\end{cases}$$

6

(c) Find the mean and standard deviation from the following data :

Class Interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	2	5	7	13	21	16	8	03

6

(d) Show that $\overline{|n+1} = n\overline{|n}$. 2

4. (a) Find the Fourier series of the function $f(x)=e^{-2x}$ in the interval $-\pi<x<\pi$.

7

(b) Determine the half-range Fourier sine series for the function

$$f(x)=\begin{cases} x & ; \quad 0<x<\pi/2 \\ \pi-x & ; \quad \pi/2<x<\pi \end{cases}$$

6

(c) Show that : 3+4=7

(i) $\Gamma\frac{1}{2}=\sqrt{\pi}$

(ii) $\beta\left(p, \frac{1}{2}\right)=2^{2p-1}\times\beta(p, p)$

5. (a) If $\vec{a}$ is a constant vector, show that

$$curl\left(\vec{r}\times\vec{a}\right)=-2\vec{a}$$

where $\vec{r}=x\hat{i}\times y\hat{j}+z\hat{k}$. 2

(b) (i) If $\vec{r}=\sin t\,\hat{i}+\cos t\,\hat{j}+t\hat{k}$, find

$$\left|\frac{d^2\vec{r}}{dt^2}\right|.$$

(ii) If $\vec{a}$ has a constant length, then prove that $\vec{a}$ and $\frac{d}{dt}\vec{a}$ are perpendicular provided that $\frac{d\vec{a}}{dt} \neq 0$.

(iii) Determine the constant 'a' so that the vector

$\vec{v} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+az)\hat{k}$

is solenoidal.

(iv) If $\vec{a}$ is a differentiable vector function of the scalar variable 't', then prove that

$$\frac{d}{dt}\left(\vec{a}\times\frac{d\vec{a}}{dt}\right) = \vec{a}\times\frac{d\vec{a}}{dt}.$$

2×4=8

(c) Reduce the matrix A to its normal form

$$\text{where } A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

and hence find its rank.

6+1=7

(d) Reduce the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ to its row echelon form and find the rank of A. 2+1=3

6. (a) Find the Fourier series for the function $f(x) = 2 + x^2$ in $-1 < x < 1$. 7

(b) Two urn, similar in appearance contain following numbers of white and black balls

Urn I : 6 white and 4 black balls

Urn II : 5 white and 5 black balls

One urn is selected at random and a ball is drawn from it. It happens to be white. What is the probability that it has come from the first urn? 6

(c) Find the inverse of the following matrix

$$\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 1 & 0 \end{pmatrix}$$ 7