53 (MA 201) ENMA–II

2019

ENGINEERING MATHEMATICS–II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* If $u = \frac{y^2}{2x}$ and $v = \frac{x^2 + y^2}{2x}$, then find $\frac{\partial(u, v)}{\partial(x, y)}$. 4

(b) Show that $\lceil \frac{1}{2} = \sqrt{x}$. 6

Contd.

(c) Find the Fourier series for the function defined as

$$f(x)=\begin{cases}\pi x, & 0\le x<1\\ \pi(2-x), & 1\le x\le 2\end{cases}$$

Hence, deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}$$ 8+2=10

2. (a) Find median and mode from the following data: 3+3=6

Class interval	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	3	6	18	10	5

(b) If two dice are thrown together, find the probability of getting either at least one 6 or a sum of 8. 4

(c) The function $f(x)$ is given as

$$f(x)=\begin{cases}x, & 0<x\le 1\\ \frac{3-x}{4}, & 1<x\le 3\\ 0, & \text{elsewhere}\end{cases}$$

Can $f(x)$ be a probability density function? If so, find the distribution function. 4+6=10

3. (a) If $u=x\sqrt{1-y^2}+y\sqrt{1-x^2}$ and $v=\sin^{-1}x+\sin^{-1}y$, show that u and v are functionally related. Also, find their relationship. 2+2=4

(b) Find the Fourier series for the function $f(x)=x$ in the interval $(-\pi,\pi)$. 7

(c) Show that,

$$\beta(m+1,n)+\beta(m,n+1)=\beta(m,n)$$ 4

(d) If X is a random variable with probability mass function $P(X=x)=q^x p$; $x=0,1,2,\ldots\ldots,\infty$, $p+q=1$, find the moment generating function of X and hence find $E(X)$. 2+3=5

4. (a) Show that any square matrix can be uniquely expressible as sum of a symmetric matrix and a skew-symmetric matrix. 5

(b) Using Green's theorem, evaluate

$$\oint_C (xy+y^2)\,dx+x^2\,dy$$

where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. 5

(c) Reduce the following matrix to normal form : 7

$$\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

(d) Show that the matrix 3

$$\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$$ is nilpotent of order 3.

5. (a) Evaluate by Stokes' theorem

$\oint_C \left(e^x dx + 2y dy - dz\right)$,

where C is the curve $x^2 + y^2 = 4, z = 2$. 5

(b) If $F = 3xy\,\hat{i} - y^2\,\hat{j}$, evaluate $\int_C F \cdot dr$, where C is the curve in the xy-plane $y = 2x^2$ from $(0, 0)$ to $(1, 2)$. 5

(c) Reduce the following matrix to its row echelon form and hence find its rank. 6+1=7

$$\begin{pmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{pmatrix}$$

(d) If A is real skew-symmetric matrix such that $A^2 + I = 0$, show that A is orthogonal. 3

6. (a) Find the inverse of the following matrix : 7

$$\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 1 & 0 \end{pmatrix}$$

(b) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q is the point $(5, 0, 4)$. 4

(c) Find the unit tangent vector to any point on the curve $x = a\cos t$, $y = a\sin t$, $z = bt$. 4

(d) The acceleration of a point at any time $t \geq 0$ is given by

$$a = \frac{dv}{dt} = 12\cos 2t\,\hat{i} - 8\sin 2t\,\hat{j} + 16t\,\hat{k}.$$

If the velocity v and displacement r are zero at $t = 0$, find v and r at any time. 5