53 (MA 201) ENMA-II

2019

ENGINEERING MATHEMATICS–II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***any five*** *questions.*

1. *(a)* Show that $\beta\left(m, \frac{1}{2}\right) = 2^{2m-1} \cdot \beta(m, m)$. 7

(b) Find the Fourier series of the function $f(x) = x$ in the interval $-\pi < x < \pi$. 7

(c) If u, v are functions of r, s and r, s are functions of x, y, then prove that

$$\frac{\partial(u, v)}{\partial(x, y)} = \frac{\partial(u, v)}{\partial(r, s)} \times \frac{\partial(r, s)}{\partial(x, y)}.$$ 6

Contd.

2. *(a)* Prove that "A real 2×2 normal matrix is either symmetric or the sum of a scalar matrix and a skew-symmetric matrix". 5

(b) Determine the values of α, β, γ, when

$$A = \begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix}$$ is orthogonal.

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(c) Reduce the matrix A to its normal form when $A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$ and hence find the rank of A. 5

(d) Compute the inverse of

$$A = \begin{pmatrix} -1 & 0 & -2 & 1 \\ -2 & 1 & 0 & 1 \\ 1 & 0 & 2 & -1 \\ -4 & 1 & -3 & 1 \end{pmatrix}$$ by elementary row transformation. 6

3. *(a)* If X is a random variable having probability density function

$$f(x) = \begin{cases} x & ; \text{ when } 0 < x < 1 \\ 2 - x & ; \text{ when } 1 \le x < 2 \\ 0 & ; \text{ elsewhere} \end{cases}$$

then determine μ_1' and μ_2. 4+4=8

(b) If X is a random variable which takes the values $X_k = \frac{(-1)^k \cdot 2^k}{k} (k = 1, 2, 3, \ldots)$ with $P_k = \frac{1}{2^k}$, then find $E(X)$. 7

(c) If $P(X = x) = {}^nC_x \, p^x \cdot q^{n-x}$; $x = 0, 1, 2, \ldots n$, then find the moment generating function. 5

4. *(a)* Find the Fourier series of

$$f(x) = \begin{cases} -\pi & ; \ -\pi < x < 0 \\ x & ; \ 0 < x < \pi \end{cases}$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$ 6+2=8

(b) Find median and mode from the following data : 3+3=6

Class interval :	0-10	10-20	20-30	30-40	40-50
Frequency :	5	6	8	30	10

(c) Show that $\lceil n+1 = n\lceil n$. 3

(d) Three dice are thrown together. What is the probability that the same number appears on exactly two of the 3 dice? 3

5. (a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$. 5

(b) If $f(x, y, z) = 3x^2y - y^3z^2$, find *grad f* at the point $(1, -2, -1)$. 5

(c) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2i - j - 2k$. 5

(d) Show that the vector $V = (\sin y + z)i + (x\cos y - z)j + (x - y)k$ is irrational. 5

6. (a) Find the row echelon form of the matrix

$$A = \begin{pmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

and hence find its rank. 6

(b) If A is real skew-symmetric matrix such that $A^2 + I = 0$, show that A is orthogonal and is of even order. 2+3=5

(c) Evaluate by Green's Theorem $\oint_C (\cos x \sin y - xy)\,dx + \sin x \cos y\,dy$ 5

(d) Evaluate by Stokes' theorem — $\oint_C (yz\,dx + xz\,dy + xy\,dz)$ where C is the curve $x^2 + y^2 = 1, z = y^2$. 4