Total number of printed pages-7

### 53 (MA 201) ENMA-II

## 2017

#### **ENGINEERING MATHEMATICS-II**

Paper : MA 201 Full Marks : 100

Time : Three hours

# The figures in the margin indicate full marks for the questions.

If  $\bar{\sigma}$  is a differentiable vector function

Answer any five questions.

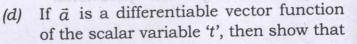
1. (a) Show that any real square matrix can be uniquely expressible as sum of a symmetric and a skew-symmetric matrix. 5

# (b) Find the inverse of the following matrix by elementary row transformations

$$X = \begin{pmatrix} 2 & 1 & -1 \\ 7 & 1 & -2 \\ 0 & 2 & 3 \end{pmatrix}$$

(c) Define eigenvalue and eigenvector of a matrix. Using Cayley-Hamilton theorem find the inverse of the matrix

$$A = \begin{pmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{pmatrix}$$
 1+1+4=6



$$\frac{d}{dt}\left(\vec{a}\times\frac{d\vec{a}}{dt}\right) = \vec{a}\times\frac{d^{2}\vec{a}}{dt^{2}}$$
 3

2. (a) Find  $\vec{\nabla}\phi$  if (i)  $\phi = \log |\vec{r}|$ (ii)  $\phi = \frac{1}{r}$ 

be unquely expressible as sum of a

4+4=8

where  $r = |\vec{r}|$ 

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(b) If  $\vec{a}, \vec{b}$  are constant vectors, w is a constant and  $\vec{r}$  is a vector function of the scalar variable 't' is given by

 $\vec{r} = \cos wt \vec{a} + \sin wt \vec{b}$ 

show that (i)  $\frac{d^2\vec{r}}{dt^2} + w^2\vec{r} = 0$ (ii)  $\vec{r} \times \frac{d\vec{r}}{dt} = w(\vec{a} \times \vec{b})$ 

4+4=8

(c) Prove that 
$$\frac{\partial(u, v, w)}{\partial(x, y, z)} \times \frac{\partial(x, y, z)}{\partial(u, v, w)} = 1$$

3. (a) Show that the Fourier series corresponding to  $f(x) = x^2$  in  $-\pi < x < \pi$  is

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$$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$$

Hence show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \text{ to } \infty$$
5+3

5+3=8

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3 Contd.

- (b) Assume that each child that is born in equally likely to be a boy or a girl. If a family has two children, then what is the probability that
  - (i) They will both be boys given that at least one is a boy
  - (ii) They will both be boys given that the eldest is a boy. 4+4=8
  - (c) Compute the quartile deviation from the following data 4

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of : Students		12	16	14	10	8	17	5

 (a) Define rank and nullity of a matrix. Find the rank and nullity of the following matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 7 & 0 \\ -4 & -2 & -5 \end{pmatrix}$$
 1+1+4=6

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4

(b) If  $\vec{F} = xy\hat{i} + (x^2 + y^2)\hat{j}$ , evaluate  $\int_C \vec{F} \cdot d\vec{r}$ , where C is the curve  $y = 2x^2$ from (2, 0) to (4, 12). 5

(c) Evaluate the distribution function F(x)for the following probability density function and calculate F(3)

$$f(x) = \begin{cases} \frac{x}{3} & ; \quad 0 < x \le 1\\ \frac{5}{27}(4-x) & ; \quad 1 < x \le 4\\ 0 & ; \quad \text{otherwise} \end{cases}$$

8+1=9

(a) Reduce the matrix  $A = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 4 & 1 & -1 & 0 \\ 2 & 1 & 3 & 3 \\ 0 & 2 & 1 & -1 \end{pmatrix}$ 5.

to its echelon form. Hence find its rank. 5+1=6

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(b) If 
$$\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$$
,

evaluate  $\iiint_V \vec{\nabla} \cdot \vec{F} \, dV$ 

where V is the region bonded by x = 0, y = 0, z = 0 and 2x + 2y + z = 4.

6

(c) Obtain the moment generating function of the random variable X having probability density function

$$f(x) = \begin{cases} x & ; & 0 < x < 1 \\ 2 - x & ; & 1 \le x < 2 \\ 0 & ; & \text{elsewhere} \end{cases}$$

Also determine  $\mu_2$ . 5+3=8

6. (a) Show that any 2×2 normal matrix is either symmetric or sum of a scalar and a skew-symmetric matrix. 5

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(b) A continuous distribution of a random variable X in the range (-3, 3) is defined by

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & ; & -3 \le x < -1\\ \frac{1}{16}(6-2x^2) & ; & -1 \le x < 1\\ \frac{1}{16}(3-x)^2 & ; & 1 \le x \le 3 \end{cases}$$

Is the function f(x) is a probability density function? Verify.

Also show that the mean is zero. 3+7=10

(c)

Find the variance of a Binomial distribution. 5

500