53 (MA 201) ENMA–II

2017

ENGINEERING MATHEMATICS–II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Show that any real square matrix can be uniquely expressible as sum of a symmetric and a skew-symmetric matrix. 5

(b) Find the inverse of the following matrix by elementary row transformations 6

$$X = \begin{pmatrix} 2 & 1 & -1 \\ 7 & 1 & -2 \\ 0 & 2 & 3 \end{pmatrix}$$

(c) Define eigenvalue and eigenvector of a matrix. Using Cayley-Hamilton theorem find the inverse of the matrix

$$A = \begin{pmatrix} 9 & 7 & 3 \\ 5 & -1 & 4 \\ 3 & 4 & 1 \end{pmatrix}$$ 1+1+4=6

(d) If $\vec{a}$ is a differentiable vector function of the scalar variable 't', then show that

$$\frac{d}{dt}\left(\vec{a} \times \frac{d\vec{a}}{dt}\right) = \vec{a} \times \frac{d^2\vec{a}}{dt^2}$$ 3

2. (a) Find $\vec{\nabla}\phi$ if (i) $\phi = log\,|\vec{r}\,|$

(ii) $\phi = \frac{1}{r}$

where $r = |\vec{r}\,|$ 4+4=8

(b) If $\vec{a}, \vec{b}$ are constant vectors, w is a constant and $\vec{r}$ is a vector function of the scalar variable 't' is given by

$$\vec{r} = \cos wt\, \vec{a} + \sin wt\, \vec{b}$$

show that (i) $\frac{d^2\vec{r}}{dt^2} + w^2\vec{r} = 0$

(ii) $\vec{r} \times \frac{d\vec{r}}{dt} = w\left(\vec{a} \times \vec{b}\right)$

4+4=8

(c) Prove that $\frac{\partial(u, v, w)}{\partial(x, y, z)} \times \frac{\partial(x, y, z)}{\partial(u, v, w)} = 1$

4

3. (a) Show that the Fourier series corresponding to $f(x) = x^2$ in $-\pi < x < \pi$ is

$$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$$

Hence show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots \text{ to } \infty$$

5+3=8

(b) Assume that each child that is born in equally likely to be a boy or a girl. If a family has two children, then what is the probability that

(i) They will both be boys given that at least one is a boy

(ii) They will both be boys given that the eldest is a boy. 4+4=8

(c) Compute the quartile deviation from the following data 4

Marks :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of : Students	10	12	16	14	10	8	17	5

4. (a) Define rank and nullity of a matrix. Find the rank and nullity of the following matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 7 & 0 \\ -4 & -2 & -5 \end{pmatrix}$$ 1+1+4=6

(b) If $\vec{F} = xy\hat{i} + (x^2 + y^2)\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $y = 2x^2$ from (2, 0) to (4, 12). 5

(c) Evaluate the distribution function $F(x)$ for the following probability density function and calculate $F(3)$

$$f(x) = \begin{cases} \dfrac{x}{3} & ; \quad 0 < x \le 1 \\ \dfrac{5}{27}(4 - x) & ; \quad 1 < x \le 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

8+1=9

5. (a) Reduce the matrix $A = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 4 & 1 & -1 & 0 \\ 2 & 1 & 3 & 3 \\ 0 & 2 & 1 & -1 \end{pmatrix}$ to its echelon form. Hence find its rank.

5+1=6

(b) If $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\,\hat{j} - 4x\hat{k}$,

evaluate $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV$

where V is the region bonded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. 6

(c) Obtain the moment generating function of the random variable X having probability density function

$$f(x) = \begin{cases} x & ; \quad 0 < x < 1 \\ 2 - x & ; \quad 1 \le x < 2 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

Also determine μ_2. 5+3=8

6. (a) Show that any 2×2 normal matrix is either symmetric or sum of a scalar and a skew-symmetric matrix. 5

(b) A continuous distribution of a random variable X in the range $(-3, 3)$ is defined by

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & ; \quad -3 \le x < -1 \\ \frac{1}{16}\left(6-2x^2\right) & ; \quad -1 \le x < 1 \\ \frac{1}{16}(3-x)^2 & ; \quad 1 \le x \le 3 \end{cases}$$

Is the function $f(x)$ is a probability density function ? Verify.

Also show that the mean is zero.

3+7=10

(c) Find the variance of a Binomial distribution. 5
