2014

ENGINEERING MATHEMATICS–II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. (a) (i) Evaluate : $\dfrac{d^2}{dt^2}\left[\vec{r}\;\dfrac{d\vec{r}}{dt}\;\dfrac{d^2\vec{r}}{dt^2}\right]$

(ii) Find $\vec{r}$ from the equation

$$\frac{d^2\vec{r}}{dt^2} = \vec{a}t + \vec{b},$$

where $\vec{a}$ and $\vec{b}$ are constant vectors and given that both $\vec{r}$ and $\dfrac{d\vec{r}}{dt}$ vanish when $t = 0$. 3+3=6

Contd.

(b) Expand $f(x)=\sqrt{1-\cos x}$ as a Fourier series in the interval $0<x<2\pi$. Hence evaluate

$$\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+\cdots\text{to }\infty=\frac{1}{2}$$ 8

(c) Reduce the following matrix to its canonical form

$$A=\begin{pmatrix}-1 & 0 & -2 & 1\\ -2 & 1 & 0 & 1\\ 1 & 0 & 2 & -1\\ -4 & 1 & -3 & 1\end{pmatrix}$$

Find its rank and nullity. 5+1=6

2. (a) Find the mean and standard deviation from the following data :

Class Interval	0–5	5–10	10–15	15-20	20–25	25–30	30–35	35–40
Frequency	2	5	7	13	21	16	8	3

2+4=6

(b) Find the half-range cosine series for the function $f(x)=(x-1)^2$ in the interval $0<x<1$. Hence show that

$$\pi^2=8\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\text{to }\infty\right)$$ 6

(c) Find $\vec{\nabla}\phi$ if (i) $\phi = log|\vec{r}|$,

(ii) $\phi = \frac{1}{r}$, $r = |\vec{r}|$ 2+2=4

(d) State and prove Baye's theorem. 1+3=4

3. (a) Define involutory and idempotent matrix. Prove that any involutory matrix is expressible as a difference of idempotent matrices. 2+3=5

(b) Find the Fourier series for the function

$$f(t) = \begin{cases} -1 & \text{for } -\pi < t < -\pi/2 \\ 0 & \text{for } -\pi/2 < t < \pi/2 \\ 1 & \text{for } \pi/2 < t < \pi \end{cases}$$ 5

(c) The function $f(x)$ is given as follows :

$$f(x) = \begin{cases} x & \text{for } 0 < x \le 1 \\ \frac{3-x}{4} & \text{for } 1 < x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

Can $f(x)$ be a probability function ? If so, find the distribution function. 2+3=5

(d) Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = (3x^2 + 6y)\hat{i}$ $-14yz\hat{j} + 20xz^2\hat{k}$ and C is the straight line joining $(0, 0, 0)$ and $(1, 1, 1)$. 5

4. (a) If $A = \begin{pmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{pmatrix}$,

then what is the rank of A^{-1} ? 5

(b) If $u = 3x + 2y - z$, $v = x^2 + y^2 + z^2$ and $w = x(x + 2y - z)$, show that they are functionally related and find the relation. 5

(c) If $\vec{f} = x^2 y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$, find $div.\vec{f}$, $curl\vec{f}$ and $curl\left(curl\vec{f}\right)$ 2+2+2=6

(d) If X is a random variable with probability mass function $P(X = x) = q^x p$; $x = 0, 1, 2, \cdots\infty$, find the moment generating function of X and hence find $E(X)$. 2+2=4

5. *(a)* Prove that

(i) $\beta(m, 1/2) = 2^{2m-1}\beta(m, m)$

(ii) $\Gamma(m)\Gamma(m+1/2) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

3+3=6

(b) If $\vec{r} = \vec{a}\cos wt + \vec{b}\sin wt$, where $\vec{a}$, $\vec{b}$ are constant vectors and w is a constant,

show that *(i)* $\frac{d^2\vec{r}}{dt^2} + w^2\vec{r} = 0$

(ii) $\vec{r} \times \frac{d\vec{r}}{dt} = w\left(\vec{a} \times \vec{b}\right)$ 2+2=4

(c) State the conditions under which a Poisson distribution can be derived from Binomial distribution. Also show that if X follows Poisson distribution with parameter λ, then $E(X) = V(X) = \lambda$. 2+4=6

(d) Find the volume V of the parallelopiped P in IR^4 determined by the vectors

$$U_1 = (2, -1, 4, -3),\ U_2 = (-1, 1, 0, 2),$$
$$U_3 = (3, 2, 3, -1),\ U_4 = (1, -2, 2, 3)$$ 4

6. (a) Show that $\Gamma(1/2) = \sqrt{\pi}$. 5

(b) Find the inverse of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

by using Cayley-Hamilton theorem. 5

(c) If X is a random variable with moment generating function $M_x(t) = \frac{1}{2}\left(e^{at} + b\right)$, find the values of a and b. Given that moment of X is μ. Also find its variance. 2+2=4

(d) If $\vec{F} = Z\hat{i} - x\hat{j} + y^2\hat{k}$, evaluate $\iiint_v \vec{F} dv$ where v is the region bounded by the planes $x = y = z = 0$, $x = y = z = 1$. 6

7. *(a)* *(i)* If $\phi = 2x^3y^2z^4$, find $div\left(grad\ \phi\right)$

(ii) Evaluate :

$$\iint (x+z)dydz + (y+z)dzdx + (x+y)dxdy$$

where S is the surface of the sphere

$x^2 + y^2 + z^2 = 4$. 4+4=8

(b) A continuous distribution of a variable X in the range $(-3,\ 3)$ is defined by

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2, -3 < x \le -1 \\ \frac{1}{16}\left(2-6x^2\right), -1 < x \le 1 \\ \frac{1}{16}(3-x)^2, 1 < x < 3 \end{cases}$$

verify that the area under the curve is unity. Show that the mean is zero. 2+2=4

(c) If $P(x) = \begin{cases} \frac{x}{15}, x = 1, 2, 3, 4, 5 \\ 0\ ,\ \text{elsewhere} \end{cases}$

Find *(i)* $P\ [X = 1\ or\ 2]$