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53 (MA 201) ENMA-II

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(May)

ENGINEERING MATHEMATICS–II

Paper : MA 201

Full Marks : 100

Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* If $\vec{a}$, $\vec{b}$ are constant vectors, w is a constant and $\vec{r}$ is a vector point function of the scalar variable t given by

$\vec{r} = \cos wt\,\vec{a} + \sin wt\,\vec{b}$, show that

(i) $\dfrac{d^2\vec{r}}{dt^2} + w^2\vec{r} = 0$

(ii) $\vec{r} \times \dfrac{d\vec{r}}{dt} = w\left(\vec{a} \times \vec{b}\right)$ 4+2=6

(b) Find the median and mode of the following distribution : 3+3=6

Wages (in Rs.)	:	20-30	30-40	40-50	50-60	60-70
No. of labourers	:	3	5	20	10	5

(c) (i) Prove that the product of the characteristic roots of a square matrix of order n is equal to the determinant of the matrix. 3

(ii) Find the inverse of the following matrix using Cayley-Hamilton theorem :

$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$ 5

2. (a) Show that for Poisson distribution, mean and variance are equal. 3+3=6

(b) If a function 'f' is bounded periodic with period 2π and integrable on $[-\pi, \pi]$ and piecewise monotonic on $[-\pi, \pi]$, then prove that

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n \cos nc_0+b_n \sin nc_0)$$

$$=\begin{cases} \frac{f(c_0-)+f(c_o+)}{2} & ; -\pi < c_0 < \pi \\ \frac{1}{2}\{f(\pi-)+f(-\pi+)\} & ; c_0 = \pm\pi \end{cases}$$

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(c) Compute the inverse of the following matrix by elementary transformation

$$A=\begin{pmatrix} 1 & 1 & 1 & 1 \\ -2 & 1 & -1 & -2 \\ -4 & -2 & -3 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

6

3. (a) (i) Let X have the p.d.f.

$$f(x)=\begin{cases} \frac{1}{2}e^{-x/2} & , x>0 \\ 0 & , \text{otherwise} \end{cases}$$

Find its m.g.f. and hence find its mean and variance. 2+2+1=5

(ii) If $A_1, A_2, \ldots\ldots\ldots, A_n$ are n mutually exclusive events, then show that the probability of occurrence of either A_1 or A_2 or .... or A_n is

$$P(A_1 \cup A_2 \cup \ldots\ldots \cup A_n) = P(A_1) + P(A_2) + \ldots\ldots + P(A_n).$$

5

(b) Find $\vec{\nabla}\phi$ and $\left|\vec{\nabla}\phi\right|$ where $\phi = r^2 e^{-r}$ and $r = |\vec{r}|$. 5+1=6

(c) Reduce the matrix $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{pmatrix}$

to its row cannonical form. 4

4. (a) (i) Prove that 3

$$2^n \Gamma(n + ½) = 1 \cdot 3 \cdot 5 \cdot \ldots\ldots\ldots\ldots (2n-1) \cdot \sqrt{\pi}$$

(ii) Prove that 4

$$\Gamma(n)\Gamma\left(\frac{1-n}{2}\right) = \frac{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}\right)}{2^{1-n}\cos\frac{n\pi}{2}}, \; 0 < n < 1,$$

(*b*) Reduce the matrix

$$A = \begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

to its normal form and hence find the rank of A.

4+1=5

(*c*) (*i*) In a college 25% of the students failed in mathematics, 15% of the students failed in chemistry and 10% failed in both. A student is selected at random. What is the probability that

(i) he failed in mathematics given that he failed in chemistry.

(ii) he failed in mathematics or in chemistry.

2+2=4

(*ii*) Let n dice be thrown. Find the mathematical expectation of the (*i*) sum and (*ii*) product of the points on them.

2+2=4

5. *(a)* *(i)* If A is a square matrix and $A-\frac{1}{2}I$ and $A+\frac{1}{2}I$ are orthogonal, prove that $A^2=\frac{3}{4}I$ and A is skew-symmetric.

1+2=3

(ii) If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are the eigen values of a matrix A, then find the eigen values of adj. A. 2

(b) Evaluate the distribution function $F(x)$ for the following p.d.f. and Calculate $F(2)$.

$$f(x)=\begin{cases}\frac{x}{3} & ; \ 0<x\le 1\\ \frac{5}{27}(4-x) & ; \ 1<x\le 4\\ 0 & ; \ \text{elsewhere}\end{cases}$$

4+1=5

(c) *(i)* Show that

$$\Gamma\left(\tfrac{3}{2}-x\right)\Gamma\left(\tfrac{3}{2}+x\right)=\pi\left(\tfrac{1}{4}-x^2\right)\sec\pi x,$$

provided $-1<2x<1$. 4

(ii) Evaluate $\int_c \vec{F}\cdot d\vec{r}$, where $\vec{F}=\left(3x^2+6y\right)\hat{i}-14yz\hat{j}+20xz^2\hat{k}$ and c is the straight line joining (0, 0, 0) and (1, 1, 1). 6

6. (a) Find a Fourier series for $f(x)$ in the interval $(-\pi, \pi)$, where

$$f(x)=\begin{cases}\pi+x & ; \quad -\pi<x<0 \\ \pi-x & ; \quad 0<x<\pi\end{cases}$$ 6

(b) Evaluate $\iiint_V (2x+y)\,dv$, where v is the close region bounded by the cylinder $z=4-x^2$ and the planes $x=0$, $y=0$, $y=2$ and $z=0$. 7

(c) (i) If a matrix X is real, skew-symmetric and $X^2+I=0$, then prove that X is of even order. 2

(ii) If $\begin{pmatrix} -1/3 & x & y \\ x & -1/3 & y \\ y & x & -1/3 \end{pmatrix}$ is orthogonal,

find x and y. 2

(iii) Evaluate $\Gamma\left(-1/2\right)$. 3

7. (a) (i) If $\phi=2x^3y^2z^4$, prove that

$div(grad\,\phi)=12xy^2z^4+4x^3z^4+24x^3y^2z^2$. 3

(ii) If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} + (-x - y)\hat{k}$, prove that $\vec{F} \cdot Curl\ \vec{F} = 0$. 3

(iii) If $u = 3x^2y$ and $v = xz^2 - 2y$, prove that grad $(grad\, u \cdot grad\, v) =$
$(16xz^2 - 12x)\hat{i} + 6xz^2\hat{j} + 12xyz\hat{k}$.
4

(b) If
$$u = xy + yz + zx$$
$$v = x^2 + y^2 + z^2$$
$$w = x + y + z$$
Determine whether there is a functional relationship among u, v, w and if so find it.
7

(c) If
$$y_1 = \cos x_1$$
$$y_2 = \sin x_1 \cos x_2$$
$$y_3 = \sin x_1 \sin x_2 \cos x_3$$
then find $J\left(\frac{y_1, y_2, y_3}{x_1, x_2, x_3}\right)$. 3
