Total number of printed pages–6

53 (MA 201) ENMA-II

2017

ENGINEERING MATHEMATICS-II

Paper : MA 201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Calculate the median and mode from the following data :

Class Interval :	0–10	10–20	20–30	30–40	40–50
Frequency :	7	9	5	30	10

3+3=6

(b) Find the Fourier series of the function $f(x)=x+x^2, -\pi<x<\pi$. Also show that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-.....$

6+2=8

Contd.

(c) If $\vec{a}$ is a constant vector, then prove that $div(\vec{r}\times\vec{a})=0$ and $curl(\vec{r}\times\vec{a})=-2\vec{a}$. 4

(d) If

$$\vec{A}=\left(2x^3y-x^5\right)\hat{i}+\left(e^{xy}-y^2\sin x\right)\hat{j}+y^2.\cos x\,\hat{k},$$

then find $\dfrac{\partial\vec{A}}{\partial x}$. 2

2. (a) If $u_1=\dfrac{x_2x_3}{x_1}$, $u_2=\dfrac{x_3x_1}{x_2}$ and $u_3=\dfrac{x_1x_2}{x_3}$, then prove that $J(u_1,u_2,u_3)=4$ 5

(b) Show that

$$\beta(m,n)=2\int_0^{\pi/2}(\sin\theta)^{2m-1}.(\cos\theta)^{2n-1}d\theta \quad ,$$

$m>0, n>0$ 4

(c) Determine the half range cosine series for the function 5

$$f(x)=\begin{cases}x & , 0<x<\pi/2\\ \pi-x & , \pi/2<x<\pi\end{cases}$$

(d) A pair of dice is rolled together till a sum of either 4 or 6 is obtained. Find the probability that 6 comes before 4. 6

3. (a) If $\vec{r}=a\cos t\,\hat{i}+a\sin t\,\hat{j}+bt\,\hat{k}$, find

$$\left[\frac{d\vec{r}}{dt}\;\frac{d^2\vec{r}}{dt^2}\;\frac{d^3\vec{r}}{dt^3}\right]$$ 4

(b) If μ_r and μ_r' be the r-th central moment and r-th raw moment of a random variable, then show that $\mu_2=\mu_2'-\mu_1'^2$, where $r=1,2$. 5

(c) Show that

$$\int_0^{\pi/2}\sin^p x.\cos^2 x\,dx=\frac{1}{2}.\frac{\left\lceil\frac{p+1}{2}\right.\left\lceil\frac{q+1}{2}\right.}{\left\lceil\frac{p+q+2}{2}\right.}$$

4

(d) If $V(X)$ be the variance of a random variable X, then show that for a Binomial distribution, $V(X)=npq$, where n = no. of variables, p = probability of success and q = probability of failure. 7

4. (a) If $\vec{F} = (2y+3)\hat{i} + xz\hat{j} + (yz-x)\hat{k}$, evaluate $\int_C \vec{F}.d\vec{r}$ along $x = 2t^2$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$. 6

(b) Find the moment generating function of the variable X_i such that $P(X_i = k) = Pq^k$; $\quad i = 1,2,\ldots,n$ and $k = 0,1,2,\ldots,n$. 6

(c) Prove that

$$\beta\left(n+\frac{1}{2}, n+\frac{1}{2}\right) = \frac{\left|\overline{n+\frac{1}{2}}.\sqrt{\pi}\right.}{2^{2n}.\left|\overline{n+1}\right.}$$ 4

(d) If $\vec{A} = x^2yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$, $\vec{B} = 2z\hat{i} + y\hat{j} - x^2\hat{k}$, find the value of $\frac{\partial^2}{\partial x.\partial y}(\vec{A}\times\vec{B})$ at $(1,0,-2)$. 4

5. (a) At any time t, the vector from the origin to a moving point is given by $\vec{r} = \vec{a}\cos\lambda t + \vec{b}\sin\lambda t$, where $\vec{a}$ & $\vec{b}$ are constant vectors and w is a constant. Find the velocity $\vec{v}$ and show that $\vec{r}\times\vec{v} = \lambda(\vec{a}\times\vec{b})$.

2+4=6

(b) If the matrix $\begin{bmatrix} x & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & y \\ z & s & t \end{bmatrix}$ is orthogonal, then find x, y, z, s and t. 6

(c) Prove that a real 2×2 normal matrix is either symmetric or the sum of a scalar matrix and a skew-symmetric matrix.

3

(d) Find the inverse of the following matrix by elementary row transformation

$$A = \begin{bmatrix} -1 & 0 & 2 & 1 \\ -2 & 1 & 0 & 1 \\ 1 & 0 & 2 & -1 \\ -4 & 1 & -3 & 1 \end{bmatrix}$$ 5

6. *(a)* Give an example (with justification) of a real matrix that is normal but is not symmetric, skew-symmetric or orthogonal. 2+2+2=6

(b) Prove that a vector function $\vec{a}(t)$ have constant magnitude if and only if

$\vec{a}.\frac{d\vec{a}}{dt}=0$. 5

(c) If A is real skew-symmetric matrix such that $A^2+I=0$, show that A is orthogonal and is of even order. 2+2=4

(d) Reduce the following matrix into Row Echelon form and hence find the rank of the matrix

$$A=\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$$ 2+3=5