53 (MA 201) ENMA II

2013C

(December)

ENGINEERING MATHEMATICS-II

Paper : MA-201

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* *(i)* If $\phi(x, y, z) = xy^2 z$ and 3

$\vec{A} = xz\hat{i} - xy^2\hat{j} + yz^2\hat{k}$,

find $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ at the point (2, –1, 1)

(ii) If $\vec{A} = cos(xy)\hat{i} + (3xy - 2x^2)\hat{j} - (3x + 2y)\hat{k}$

find $\frac{\partial^2 \vec{A}}{\partial x^2}, \frac{\partial^2 \vec{A}}{\partial y^2}, \frac{\partial^2 \vec{A}}{\partial x \partial y}$. 2×3=6

Contd.

(b) Two players A and B have probabilities 1/7 and 1/8 respectively to win a race. What is the probability that neither will win ? 3

(c) Find a Fourier series to represent $x - x^2$ from $x = -\pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \frac{\pi^2}{12}$$ 8

2. (a) (i) If A and B are two idempotent matrices, then under which conditions A+B will be idempotent ? 3

(ii) If $\begin{pmatrix} A^{-1} & 0 \\ X & A^{-1} \end{pmatrix} = \begin{pmatrix} A & 0 \\ B & A \end{pmatrix}^{-1}$ and A is non-singular, then find out the value of X. 3

(b) Obtain the Fourier expansion of $x \sin x$ as a cosine series in $(0, \pi)$. Hence show that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots \text{to } \infty = \frac{\pi - 2}{4}$$ 7

(c) Prove that the vector function $\vec{a}(t)$ to have constant magnitude iff $\vec{a}.\frac{d\vec{a}}{dt}=0$. 7

3. (a) (i) State and prove Baye's theorem. 2+3=5

(ii) Let X be a random variable with p.d.f.

$$f(x)=c(1-x); \quad 0<x<1.$$

Find C, $E(X)$ and $V(X)$. 1+2+2=-5

(b) A particle moves along the curve $x=4\cos t$, $y=4\sin t$, $z=6t$. Find the velocity and acceleration at time $t=\pi/2$. 5

(c) Under what condition the rank of the following matrix A is 3 ? Is it possible for the rank to be 1 ? Why ? 5

$$\begin{pmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{pmatrix}$$

4. *(a)* Reduce the matrix A to its normal form where

$$A=\begin{pmatrix} 2 & 3 & -1 & 1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

and hence find the rank of A. 6+1=7

(b) *(i)* If X is a random variable with probability mass function $P(X=x)=q^x p$; $x=0,1,2,\cdots\infty$, $q=1-p$, find m.g.f. of X and hence find $E(X)$. 2+2=4

(i) For a binomial distribution $n=10$, $p=1/2$, Find *(i)* $P(X=2)$ and *(ii)* $P(X>1)$. 1+2=3

(c) *(i)* If $y_1=\frac{x_2x_3}{x_1}$, $y_2=\frac{x_3x_4}{x_2}$, $y_3=\frac{x_1x_2}{x_3}$, show that the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 is 4.

3

(ii) If $u=x^2-y^2$, $v=2xy$ and

$x=r\cos\theta$, $y=r\sin\theta$,

find $\dfrac{\partial(u,v)}{\partial(r,\theta)}$ 3

5. *(a)* Define Beta function. Prove that

$$B(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$ 1+6=7

(b) If $\dfrac{d\vec{u}}{dt}=\vec{\omega}\times\vec{u}$, $\dfrac{d\vec{v}}{dt}=\vec{\omega}\times\vec{v}$,

Prove that $\dfrac{d}{dt}(\vec{u}\times\vec{v})=\vec{\omega}\times(\vec{u}\times\vec{v})$. 6

(c) State Cayley-Hamilton theorem. Show that

the matrix $A=\begin{pmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{pmatrix}$

Satisfies Cayley-Hamilton theorem.

1+6=7

6. *(a)* If A is real skew-symmetric matrix such that $A^2 + I = 0$. Show that A is orthogonal and is of even order. 3+3=6

(b) *(i)* If $X \sim N(100, 4)$,

find $q(92 < X < 108)$ given that

$$p\left\{\frac{X-100}{4} > 2\right\} = 0.02275.$$ 5

(ii) If X follows Binomial distribution with probability mass function

$$P(X = x) = {}^{100}C_x \left(\frac{1}{50}\right)^x \left(\frac{49}{50}\right)^{100-x},$$

$x = 0, 1, 2, \ldots\ldots, 100.$ 5

(c) If $\vec{v} = e^{xyz}\left(\hat{i} + \hat{j} + \hat{k}\right)$, find curl $\vec{v}$. 4

7. *(a)* *(i)* Let A and B be two events such that

$P(A) = 3/4$ and $P(B) = 5/8$.

Show that

(i) $P(A \cup B) \geq 3/4$

(ii) $3/8 \leq P(A \cap B) \leq 5/8$ 2+3=5

(iii) Prove that the product of the characteristic roots of a square matrix of order n is equal to the determinant of the matrix. 5

(b) Find the inverse of the following matrix by elementary transformation, 6

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 2 \end{pmatrix}$$

(c) If $F = xu + v - y$, $G = u^2 + vy + w$,

$H = zu - v + vw$,

Compute $\dfrac{\partial(F, G, H)}{\partial(u, v, w)}$ 4
