53 (MA 101) ENMA-I

2017

ENGINEERING MATHEMATICS-I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Find the n^{th} derivative of the function $y = e^{ax+b}$. 5

(b) State limit comparisn test. Examine the convergency of the following series

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n^3 + 2n^2 + 6n}$$ 2+4=6

(c) Define an ordinary differential equation. Form a differential equation from each of the following primitives :

(i) $y = ae^x + be^{-x} + c\cos x + d\sin x$, where a, b, c, d are arbitrary constants.

Contd.

(ii) $y = \alpha x + \beta x^2$, where α, β are arbitrary constants.

1+4+4=9

2. (a) State Cauchy's general principle of convergence of a series. Using it, show that the following series is convergent

$$1 + \frac{1}{3} + \frac{1}{5} + \ldots\ldots$$

2+6=8

(b) Expand e^x in power of $(x-1)$ upto four terms. 5

(c) Solve :

$$2\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + y = e^t + 1$$

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3. (a) Solve :

(i) $xdy - ydx = \sqrt{x^2 + y^2}\, dx$

(ii) $x\frac{dy}{dx} + y = y^2 \log x$

6×2=12

(b) Trace the curve $xy^2 = 4a^2(2a - x)$. 8

4. *(a)* Does the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\left(n^{1/2}+1\right)\left(n^{1/3}+1\right)}$$

converge absolutely ? Justify. 6

(b) State Leibnitz test. Show that the series

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

is convergent if $|x| \leq 1$. 2+5=7

(c) Find the equation of the sphere through the points (0,0,0), (0,1,–1), (–1,2,0) and (1,2,3). 7

5. *(a)* Prove that the equation of the plane throught the line

$$\frac{x+3}{2} = \frac{y-2}{-2} = \frac{z-1}{3}$$

parallel to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-5}{-3}$

is $3y + 2z = 8$. 6

(b) If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. 5

(c) Find the area enclosed by the loop of the curve $x^3 + y^3 = 3axy$. 5

(d) Find the differential equation corresponding to the family of curves $y = c(x-c)^2$, where c is an arbitrary constant. 4

6. (a) Find the radius of curvature at any point θ of the curve $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$. 5

(b) Find all the asymptotes of the curve

$$f(x, y) = y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1$$

6

(c) Find the reduction formula for $\int_0^{\pi/2} \sin^m x\,dx$, where m is an even natural number. 5

(d) Find orthogonal trajectories of the curve $y - \alpha x^3 = 0$, where α is an arbitrary constant. 4