Total number of printed pages–8

53 (MA 101) ENMA-1

2015

ENGINEERING MATHS-I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer question **1** *and* ***any four*** *from the rest*

1. *(a)* Choose the correct option— $6\times1=6$

(i) If $y = x\log_e x$ then $\frac{dy}{dx}$ is

(a) 1,

(b) $\log_e x - 1$,

(c) $\log_e x$,

(d) $1 + \log_e x$

(ii) The derivative of $x^2 \cos x$ is

(a) $2\cos x - x^2 \sin x$

(b) $2x\cos x - x^2 \sin x$

(c) $2x^2 \cos x - x^2 \sin x$

(d) $2x^2 \cos x - x\sin x$

Contd.

(iii) The equation of the tangent to the curve $y = x^2 + 1$ at the point (1, 2) is

(a) $y - 2 = -2(x - 1)$

(b) $y - 2 = 4(x - 1)$

(c) $y - 2 = 2(x - 1)$

(d) $y - 2 = 2(x + 1)$

(iv) The solution of $\frac{dy}{dx} = m$ represents a family of—

(a) straight lines

(b) circles

(c) parabolas

(d) hyperbolas.

(v) The centre of the sphere $x^2 + y^2 + z^2 - 4x + 5y - 6z - 1 = 0$ is

(a) $\left(2, \frac{-5}{2}, 3\right)$

(b) $\left(-2, \frac{5}{2}, -3\right)$

(c) $\left(\frac{1}{2}, \frac{-2}{5}, \frac{1}{3}\right)$

(d) None of these.

(vi) The order of $y''' + y^2 + e^{y'} = 0$ is

(a) 3, (b) 1, (c) 2, (d) Not defined.

(b) State *true* **or** *false* : 7×1=7

(i) If $y = x^4 \log_e x$ then for $n \geq 5$,

$$y_n = (-1)^{n-1} \frac{\lfloor n-5}{x^n - 4} 24$$

(ii) The radius of curvature at $\left(\frac{1}{4}, \frac{1}{4}\right)$ to the curve $\sqrt{x} + \sqrt{y} = 1$ is

$$\frac{1}{\sqrt{3}}$$

(iii) The real asymptote to the curve $y^2 = x(a^2 - x^2)$ is $x + y = 0$

(iv) Order and degree of a differential equation, if defined, are always positive integers.

(v) The differential equation corresponding to the family of curves $y = c(x - c)^2$, where c is an arbitrary constant is $(y')^3 = 4y(xy' - 2y)$

(vi) The differential equation

$y \sin 2x dx - (y^2 + \cos^2 x)dy = 0$ is exact.

7.

(vii) $(4y + 3x)dy + (y - 2x)dx = 0$ is a homogeneous differential equation.

(c) Fill in the blanks : $7 \times 1 = 7$

(i) The area bounded by the x-axis, the y-axis and the line $y - x = 1$ is ——————

(ii) $\int_0^{\pi/2} \sin^7 x dx$ equals ——————

(iii) The length of the arc of the circle $x^2 + y^2 = a^2$ lying between the ordinates $x = \frac{a}{2}$ and $x = a$ is ——

(iv) The equation of the line joining the points (4, 2, –3) and (6, –1, 2) is ——————

(v) The intercepts made on the axes by the plane $3x - 4y + 6z - 12 = 0$ are ——————

(vi) A directed line makes angles 30° and 60° with the axes of x and y respectively. The angle it makes with the z-axis is ————

(vii) The series

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots + \frac{n}{n+1} + \ldots \text{ is}$$

———— (convergent/divergent).

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2. (a) (i) If $y = e^{a \sin^{-1} x}$ prove that

$$(1 - x^2)y_2 - xy_1 - a^2y = 0.$$ 2

(ii) If $y = e^{-x}\cos x$ prove that

$$y_4 + 4y = 0.$$ 3

(iii) If $y = x \log_e \frac{x-1}{x+1}$ find y_{n-1} 3

(b) Find the asymptotes of the curve

$$x^3 - 2y^3 + 2x^2y - xy^2 + xy - y^2 + 1 = 0$$ 6

(c) Give the statement of D'Alembert's Ratio test for a series of real numbers. Is the series $\sum_n \frac{1.2.3.....n}{7.10....(3n+4)}$. 2+4=6

3. (a) Find the differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$ where A and B are arbitrary constants. 5

(b) Find the equation of the plane passing through the point (1, –2, 1) and the line of intersection of the planes 5

$2x - y + 3z - 2 = 0$ and
$x + 2y - 4z + 3 = 0$

(c) Find the equation of the sphere passing through the origin and the points $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ 5

(d) Prove that the sum of the intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the co-ordinate axes is constant. 5

4. (a) Verify Euler's theorem when

$$u = x^3 + y^3 + 3x^2y + 3xy^2$$ 5

(b) Solve **any three**— 3×5=15

(i) $xdy + ydx = 0$

(ii) $\dfrac{dy}{dx} = e^{x-y} + x^2e^{-y}$

(iii) $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$

(iv) $\dfrac{dy}{dx} = \dfrac{x+2y-3}{2x+y-3}$

(v) $\dfrac{dy}{dx} + y = e^{-x}$

5 (a) Find the orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2 + \lambda} = 1$, λ being parameter. 5

(b) Find the equation of the line through the point (3, 1, 2) to intersect the line $x + 4 = y + 1 = 2(z - 2)$ and parallel to the plane $4x + y + 5z = 0$ 5

(c) Test the convergence of the series

$$\sum_n \left(\sqrt[3]{n^3+1}-n\right)$$ 5

(d) If $I_n = \int_0^{\pi/2} x^n \sin x \, dx$

show that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$

where $n > 1$ 5

6. (a) Solve **any three** : 3×5=15

(i) $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^t$

(ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \cos 3x$

(iii) $(D^2 + 1)y = \sin 2x$

(iv) $(D^2 - 4)y = x^2$

(v) $(D^2 - 2D + 1)y = x^2e^{3x}$

(b) Show that the series $\sum_n (-1)^n \sin\left(\frac{1}{n}\right)$ is not absolutely convergent. 5

7. *(a)* Find the equation of the sphere whose centre is (2, –1, 3) and radius is 5. 2

(b) Find the area of the region bounded by the upper half of the circle $x^2+y^2 = 25$, the x-axis and the ordinates $x=-3$ and $x=4$. 5

(c) If $y = (\tan^{-1}x)^2$ show that

$$(x^2+1)^2y_2 + 2x(x^2+1)y_1 = 2$$ 3

(d) Find the radius of curvature of

$$3x^2 + xy + y^2 - 4x = 0$$ at the origin. 3

(e) Show that the curves $x^2 + 4y^2 = 8$ and $x^2 - 2y^2 = 4$ intersect orthogonally. 2

(f) Is the series $1+\frac{1}{2}+\frac{1}{2^2}+\cdots$ convergent ? 2

(g) Apply Leibnitz theorem to find y_5 when $y = e^{ax}x^3$. 3
