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53 (MA 101) ENMA-I

2013

(May)

ENGINEERING MATHEMATICS – I

Paper : MA 101

Full Marks : 100

Pass Marks : 30

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* What is the condition for two curves $f(x, y) = 0$ and $g(x, y) = 0$ to cut orthogonally ?

   Find the condition that the curves

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1$$

   shall intersect orthogonally. 1+6=7

Contd.

(b) If $I_n = \int_0^{\pi/2} x^n \sin x dx$ and $n > 1$

show that

$$I_n + n(n-1) I_{n-2} = n \cdot \left(\frac{\pi}{2}\right)^{n-1}$$

Find the value of $\int_0^{\pi/2} x^4 \sin x dx$. 6+1=7

(c) Check the convergence of the series stating the test of convergence applied

$$1 + \frac{2^2}{2^2} + \frac{2^3}{3^3} + \frac{2^4}{4^4} + \cdots + \frac{2^n}{n^n} + \cdots$$ 6

2. (a) Define order and degree of a differential equation. Find order and degree of the following differential equations

(i) $\left(\frac{d^3y}{dx^3}\right)^2 + \left(1 + \frac{dy}{dx}\right)^3 = 0$

(ii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \sin x$ 2+2+2=6

(b) Find the equation of the plane through the intersection of the planes

$x+2y+3z-4=0$ and $2x+y-z+5=0$ and perpendicular to the plane

$5x+3y+6z+8=0$ 7

(c) A plane passes through a fixed point (p, q, r) and cuts the axes in A, B, C.

Show that the locus of the centre of the sphere OABC is

$$\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=2$$ 7

3. (a) State Euler's theorem on homogeneous functions of two variables. Verify

Euler's theorem when

$$u=\sin\frac{x^2+y^2}{xy}$$ 2+5=7

(b) Find all the asymptotes of 6

$$xy^2-y^2-x^3=0$$

(c) Prove that $\frac{dy}{dx} = \frac{y^2 - 1}{x^2 - 1}$ represents rectangular hyperbolas all of which pass through the points (1,1) and (–1,–1) 7

4. (a) Find the equation of the plane passing through the points (2, 3, –3), (1, 1, –2) and (–1, 1, 4). 7

(b) Give the statement of D'Alembert's Ratio test for a series of real numbers.

Is the series

$$\frac{1.2}{3^2} + \frac{2.3}{3^3} + \frac{3.4}{3^4} + \cdots + \frac{n(n+1)}{3^{n+1}} + \cdots$$

convergent ? 2+4=6

(c) Find the length of the arc of the parabola $y^2 = 16x$ measured from the vertex to an extremity of the latus rectum. 7

5. (a) If $y = e^{m \sin^{-1} x}$ show that

(i) $(1 - x^2) y_2 - x y_1 - m^2 y = 0$

(ii) $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + m^2) y_n = 0$

Also find y_n when $x = 0$ 2+3+2=7

(b) Find the equation of the cone whose vertex is the point (1, 2, 3) and guiding curve is the circle

$$x^2+y^2+z^2=4,\ x+y+z=1$$ 7

(c) Find the distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured parallel to the line whose direction cosines are proportional to 2, 3, –6. 6

6. (a) Find the volume and surface area of the solid of revolution formed by the rotation of the parabola $y^2=4ax$ about the x-axis and bounded by the section $x=x_1$. 6

(b) Solve $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$ 4

(c) Solve :

(i) $\left(2x^2y-3y^4\right)dx+\left(3x^3+2xy^3\right)dy=0$

(ii) $y^2+\left(x-\frac{1}{y}\right)\frac{dy}{dx}=0$ 5+5=10

7. *(a)* State the limit form of the comparison test for a series of real numbers. Is the series

$$2+\frac{3}{8}+\frac{4}{27}+\cdots+\frac{n+1}{n^3}$$

convergent ? 2+3=5

(b) Solve

(i) $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$

(ii) $\frac{d^2y}{dx^2}+y=\sin 2x$ if $y=0$, $\frac{dy}{dx}=0$ when $x=0$

(iii) $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2\log x$

3×5=15
