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53 (MA 101) ENMA

2019

ENGINEERING MATHEMATICS-I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* Write the statement of the following : 2+2+2=6

   (i) Limit comparison test.

   (ii) Ratio test.

   (iii) Cauchy's root test.

   (b) Form the differential equation : ***(any two)*** 4+4=8

   (i) $y = Ae^{3x} + Be^{-x}$

   (ii) $y = Ax^2 + Bx + C$

   (iii) $y = A\cos x + B\sin x$

Contd.

(c) Expand $\sin x$ in power of $\left(x-\frac{\pi}{2}\right)$ and hence evaluate $\sin 91^\circ$, correct to four decimal places. 6

2. (a) Examine the following series : 6

$$\frac{2^p}{1^q}+\frac{3^p}{2^q}+\frac{4^p}{3^q}+\ldots\ldots,$$

where p, q are unknown.

(b) Solve the following equations : ***(any two)*** 5+5=10

(i) $\left(D^2+5D+4\right)y=3-2x$

(ii) $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$

(iii) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=5e^{3x}$

(c) Find the m^{th} derivation of $x^{m-1}\log x$. 4

3. (a) Show that the series $\sum_{n=1}^{\infty}\frac{n^3}{(n+1)!}$; $n\in\mathbb{N}$ is convergent. 6

(b) Find all the asymtotes of the curve 7

$$y^3-xy^2-x^2y+x^3+x^2-y^2-1=0$$

(c) State Leibnitz's test. Show that the alternating series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots.$ is convergent. 2+5=7

4. (a) A directed line makes angles 30° and 60° with x-axis and y-axis respectively. What angle does it make with the z-axis ? 4

(b) If $y=e^{a\sin^{-1}x}$, show that 4

$$\left(1-x^2\right)y_{n+2}-(2n-1)xy_{n+1}-\left(n^2+a^2\right)y_n=0.$$

(c) Solve the following differential equations : ***(any three)*** 4+4+4=12

(i) $\left(xy^2+x\right)dx+\left(yx^2+x\right)dy=0$

(ii) $\frac{dy}{dx}+2xy=2e^{-x^2}$

(iii) $x\frac{dy}{dx}+y=x^3y^6$

(iv) $\frac{dy}{dx}=\frac{y-x+1}{y+x+5}$

5. (a) Show that equation of the plane passing through the point (α,β,γ) and parallel to the plane $ax+by+cz+d=0$ is $a(x-\alpha)+b(y-\beta)+c(z-\gamma)=0$. 5

(b) Solve the following simultaneous equations : 5

$$\frac{dx}{dt}+5x-2y=t$$

$$\frac{dx}{dt}+2x+y=0$$

(c) Find the co-ordinates of the points, where the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ meets the plane $x-2y+3z+4=0$. 5

(d) Investigate the continuity of the following function : 5

$$f(x,y)=\begin{cases} xy\dfrac{(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y)\neq(0,0) \\ 0, & \text{if } (x,y)=(0,0) \end{cases}$$

6. (a) Find the equation of the sphere passing through the origin and the points, $A(a,0,0)$, $B(0,b,0)$ and $C(0,0,C)$. 5

(b) Find the curvature of the curve $x^2+y^3=3axy$ at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$. 5

(c) Find the volume of the solid generated by revolving the circle $x^2+y^2=a^2$ about the x-axis. 5

(d) Find the reduction formula for $\int \sin^n x\cos^m x\,dx$. 5