Total number of printed pages–5

53 (MA 101) ENMA

2019

ENGINEERING MATHEMATICS – I

Paper : MA 101

Full Marks : 100

Time : Three hours

The figures in the margin indicate full marks for the questions.

*Answer **any five** questions.*

1. *(a)* State limit comparison test. Examine the convergency of the following series.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt[3]{n}+2}$$ 2+4=6

(b) Find the equation of the sphere which passes through the points (1, 2, 3), (0, –2, 4), (4, –4, 2) and (3, 1, 4). 7

(c) Show that, every absolutely convergent series is convergent. Give an example with justification to show that the converse is not true in general. 5+2=7

Contd.

2. (a) Find the equation of the plane which contains the line $\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z}{3}$ and is perpendicular to the plane $x+y+2z=4$. 5

(b) If (l_1, m_1, n_1), (l_2, m_2, n_2) and (l_3, m_3, n_3) be the direction cosines of three mutually perpendicular lines, show that the line whose direction ratios are $l_1+l_2+l_3$, $m_1+m_2+m_3$ and $n_1+n_2+n_3$ makes equal angles with them. 6

(c) Test the following series :

$$\sum_n \left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$ 4

(d) Find the equation of the plane through the line of intersection of the planes $x+2y-3z=1$, $2x-3y+2=0$ and the origin. 5

3. (a) Expand $log_e x$ in powers of $(x-1)$ and hence evaluate $log_e(1{\cdot}1)$ correct to four decimal places. 5+1=6

(b) Find all the asymptotes of the curve

$$y^3-xy^2-x^2y+x^3+x^2-y^2-1=0$$ 6

(c) Find the centre of curvature of the curve $y=x^3-6x^2+3x+1$ at $(1, -1)$ 4

(d) Investigate the continuity of the following function :

$$f(x, y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2} & ; \text{ if } (x, y)\neq(0, 0)\\ 0 & ; \text{ if } (x, y)\neq(0, 0)\end{cases}$$

at the origin. 4

4. (a) Solve the following differential equations *(any two)*: 4×2=8

(i) $(a+x)\frac{dy}{dx}=y-ay^2$

(ii) $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$

(iii) $\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+(x+\log x-x\sin y)dy=0$

(b) If $y = e^{a \sin^{-1} x}$, show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$ 5

(c) Verify that $xdx + ydy = \dfrac{a^2 (xdy - ydx)}{x^2 + y^2}$ is exact and hence solve it. 5

(d) Apply Leibnitz theorem to $y = e^{ax}.x^5$ to find y_5. 2

5. (a) Find the radius of curvature of $\sqrt{a} = \sqrt{r} \cos\dfrac{\theta}{2}$ at (r, θ). 6

(b) Show that $\lim\limits_{\substack{x\to 0 \\ y \to 0}} xy \dfrac{x^2 - y^2}{x^2 + y^2} = 0$ 5

(c) Show that the length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $6a$. 7

(d) Evaluate : $\int_0^{2a} x^{5/2} \sqrt{2a - x}\, dx$ 2

6. (a) Solve : ***(any two)*** 4×2=8

(i) $x^2 \dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x$

(ii) $(D^2 + 6D + 9)y = 2e^{-3t}$, $D = \dfrac{d}{dt}$

(iii) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = \cos 3x$

(b) Find the volume of the solid generated by revolving the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ about the major axis. 6

(c) Solve the following simultaneous differential equation :

$$\frac{dx}{dt} + 2x - 3y = 0$$

$$\frac{dy}{dt} - 3x + 2y = 0$$ 6

(b) If $y = e^{a \sin^{-1} x}$ [illegible], show that

$(1-x^2)y_{n+2} - [illegible] - (n^2 + a^2)y_n = 0$ 5

Verify that [illegible] $\frac{a^2 (xdy - ydx)}{[illegible]}$

is exact and hence solve it.

(c) Apply [illegible] theorem [illegible]

find y_5.

5. (a) Find the radius of curvature of

$r^2 = a^2 \cos \frac{[illegible]}{2}$ at (r, θ). 5

(b) Show that [illegible] 5

(c) Show that the length of the asteroid

$x^{2/3} + y^{2/3} = a^{2/3}$ is $6a$. 7

(d) Evaluate $\int [illegible] \sqrt{2a - x}\, dx$ 2